Example

Insulated copper wire with diameter 1 mm the left end is at 20°C and the right end is at 30°C

$\dot{q} = 100 \frac{W}{m^3}$

$$A_c = \pi r^2 = \frac{\pi d^2}{4}$$

$$= 7.85 \times 10^{-7} \, m^2$$

$$K = 401 \frac{W}{mK}$$

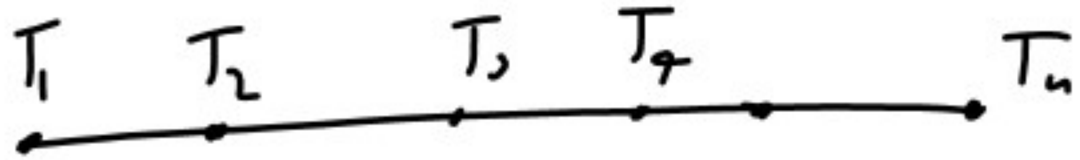
$$\nabla^2 T + \frac{\dot{q}}{K} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{K} = 0$$

$$\frac{T_{m-1} + T_{m+1} - 2T_m}{\Delta x^2} + \frac{\dot{q}}{K} = 0$$

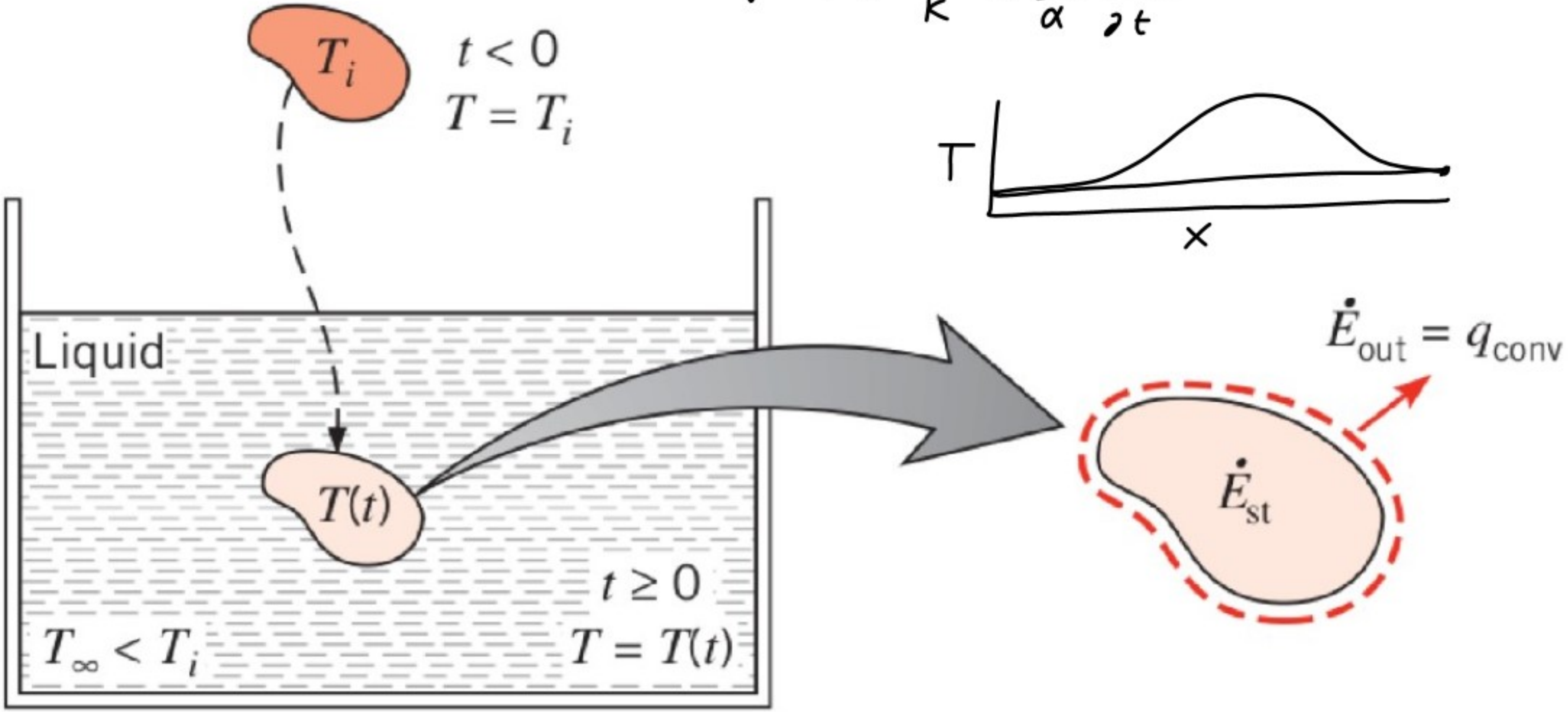
$$T_{m-1} + T_{m+1} - 2T_m = -\frac{\dot{q}}{K} A_c \Delta x$$

$$= -1.96 \times 10^{-7} \Delta x$$



$$
\begin{bmatrix}
-2 & 1 & & & \\
1 & -2 & 1 & & \\
0 & 1 & -2 & 1 & \\
 & & & & \\
 & & & 1 & -2
\end{bmatrix}
\begin{bmatrix}
T_2 \\ T_3 \\ T_4 \\ T_5 \\ \\ T_{n-2} \\ T_{n-1}
\end{bmatrix}
=
\begin{bmatrix}
1.96\times10^{-7}\Delta x - 20 \\
1.96\times10^{-7}\Delta x \\
1.96\times10^{-7}\Delta x \\
\\
1.96\times10^{-7}\Delta x - 30
\end{bmatrix}
$$

$$\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$



$$-hA_s(T-T_\infty) = \rho V c \frac{\partial T}{\partial t}$$

V volume

c specific heat

ρ density

$\theta = T - T_\infty$ $\theta_i = T_i - T_\infty$

$$\frac{\rho V c}{hA_s} \ln \frac{\theta_i}{\theta} = t$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\left(\frac{hA_s}{\rho V c}\right)t\right)$$