Steel balls 10 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 450 K in an air environment for which $T_\infty = 325$ K and $h = 25$ W/m$^2 \cdot$ K. Assuming the properties of the steel to be $k = 40$ W/m $\cdot$ K, $\rho = 7800$ kg/m^3, and $c = 600$ J/kg $\cdot$ K, estimate the time required for the cooling process.

$$\frac{\theta}{\theta_i} = e^{-t/\tau}$$

$$\tau = \frac{\rho V c}{h A_s} = \frac{7800 \cdot 0.0017 \cdot 600}{25}$$

$$= 318.24 \text{ s}$$

$$= \frac{\rho L_c c}{h}$$

$$Bi = \frac{h L_c}{k}$$

$$= \frac{25 \cdot 0.0017}{40}$$

$$= 0.001$$

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2}$$

$$= \frac{r}{3} = \frac{5\text{mm}}{3}$$

$$= 0.0017 \text{ m}$$

$$\theta = T - T_\infty = 450 - 325 = 125\ K$$

$$\theta_i = T_i - T_\infty = 1150 - 325 = 825\ K$$

$$\frac{\theta}{\theta_i} = e^{-t/\tau} \qquad \ln\frac{\theta}{\theta_i} = \frac{-t}{\tau} \qquad -\tau \ln\frac{\theta}{\theta_i} = t$$

$$-318 \ln\left(\frac{125}{825}\right) = 600\ s = 10\ min$$

Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 100-mm-thick steel plate ($\rho = 7830$ kg/m^3, $c = 550$ J/kg $\cdot$ K, $k = 48$ W/m $\cdot$ K), which is initially at a uniform temperature of $T_i = 200°C$ and is to be heated to a minimum temperature of 550°C. Heating is effected in a gas-fired furnace, where products of combustion at $T_\infty = 800°C$ maintain a convection coefficient of $h = 250$ W/m$^2 \cdot$ K on both surfaces of the plate. How long should the plate be left in the furnace?

$$\theta^* = C_1 e^{-\zeta_1^2 F_o} \cos \zeta_1 x^*$$

$$= C_1 e^{-\zeta_1^2 F_o}$$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = \frac{550 - 900}{200 - 900} = 0.917$$

$$\ln \frac{\theta^*}{C_1} = -\zeta_1^2 F_o$$

$$Bi = \frac{hL}{k} = \frac{250 \cdot 0.05}{48}$$

$$= 0.26$$

$$\zeta_1 = 0.48 \qquad C_1 = 1.038$$

$$\frac{-1}{\zeta_1^2} \ln \frac{\theta^*}{C_1} = F_o =$$