Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 100-mm-thick steel plate ($\rho = 7830 \text{ kg/m}^3$, $c = 550 \text{ J/kg} \cdot \text{K}$, $k = 48 \text{ W/m} \cdot \text{K}$), which is initially at a uniform temperature of $T_i = 200°\text{C}$ and is to be heated to a minimum temperature of 550°C. Heating is effected in a gas-fired furnace, where products of combustion at $T_\infty = 800°\text{C}$ maintain a convection coefficient of $h = 250 \text{ W/m}^2 \cdot \text{K}$ on both surfaces of the plate. How long should the plate be left in the furnace?

$$\theta^* = C_1 e^{-\zeta_1^2 F_o} \cos \zeta_1 x^*$$

$$= C_1 e^{-\zeta_1^2 F_o}$$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = \frac{550 - 800}{200 - 800} = 0.417$$

$$\ln \frac{\theta^*}{C_1} = -\zeta_1^2 F_o$$

$$Bi = \frac{hL}{k} = \frac{250 \cdot 0.05}{48}$$

$$= 0.26$$

$$\zeta_1 = 0.48 \qquad C_1 = 1.038$$

$$\alpha = \frac{k}{\rho c}$$

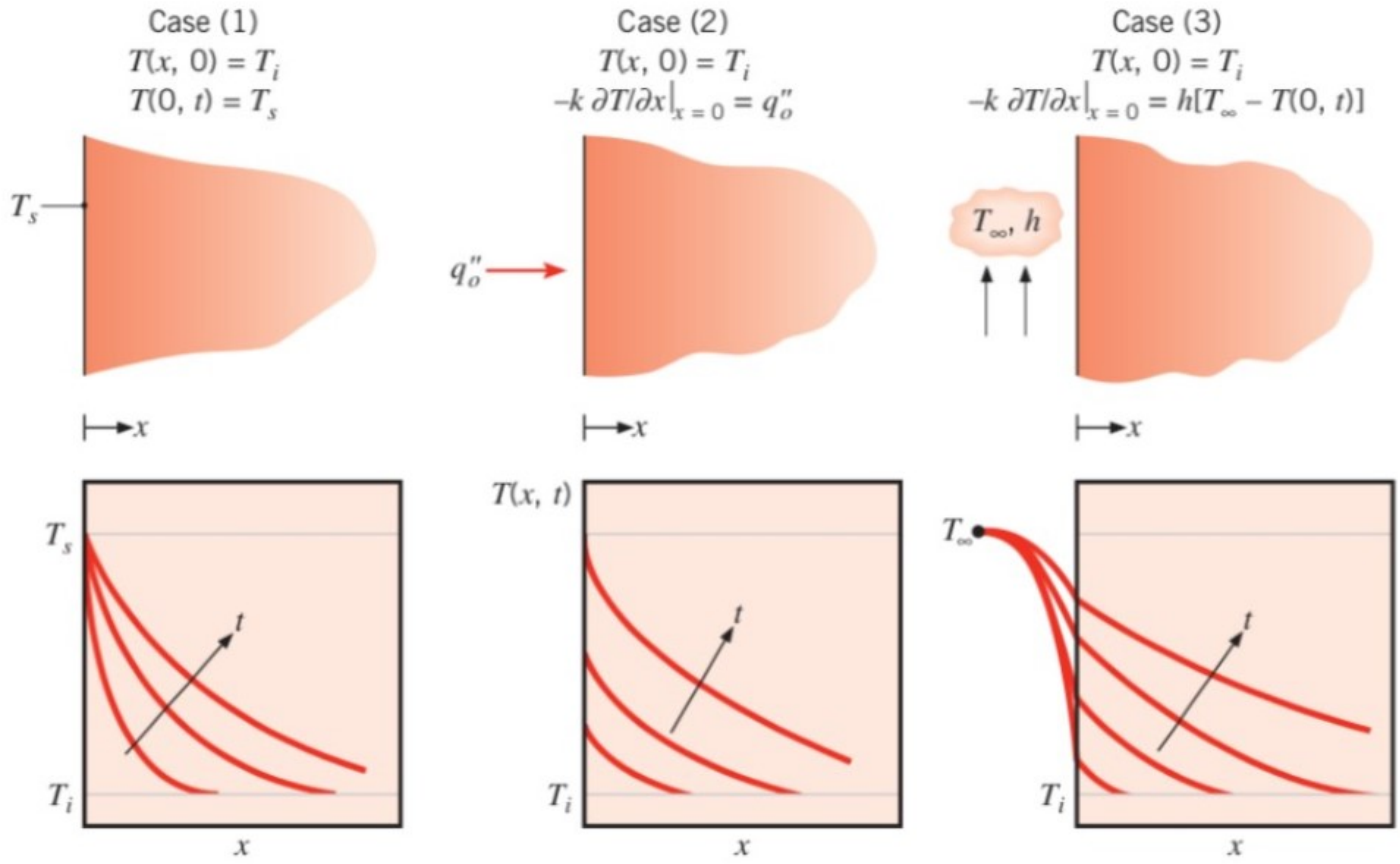
$$= \frac{48}{7830 \cdot 550}$$

$$= 1.1 \times 10^{-5}$$

$$\frac{-1}{\zeta_1^2} \ln \frac{\theta^*}{C_1} = F_o = \frac{-1}{0.48} \ln \left(\frac{0.417}{1.038} \right) = 1.9$$

$$F_o = \frac{\alpha t}{L^2} \qquad t = \frac{F_o L^2}{\alpha} = \frac{1.9 (0.05)^2}{1.1 \times 10^{-5}} = 426 \text{ s} = \boxed{7.1 \text{ min}}$$

Semi-Infinite solids



Constant Surface Temperature

$$T(0,t) = T_s$$

$$\frac{T(x,t) - T_s}{T_i - T_s} = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

$$q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi \alpha t}}$$

Error Function

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

Constant Surface Heat Flux

$$T(x,t) - T_i = \frac{2 q_s'' \sqrt{\alpha t/\pi}}{k} \exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_s'' x}{k} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Complementary Error Function

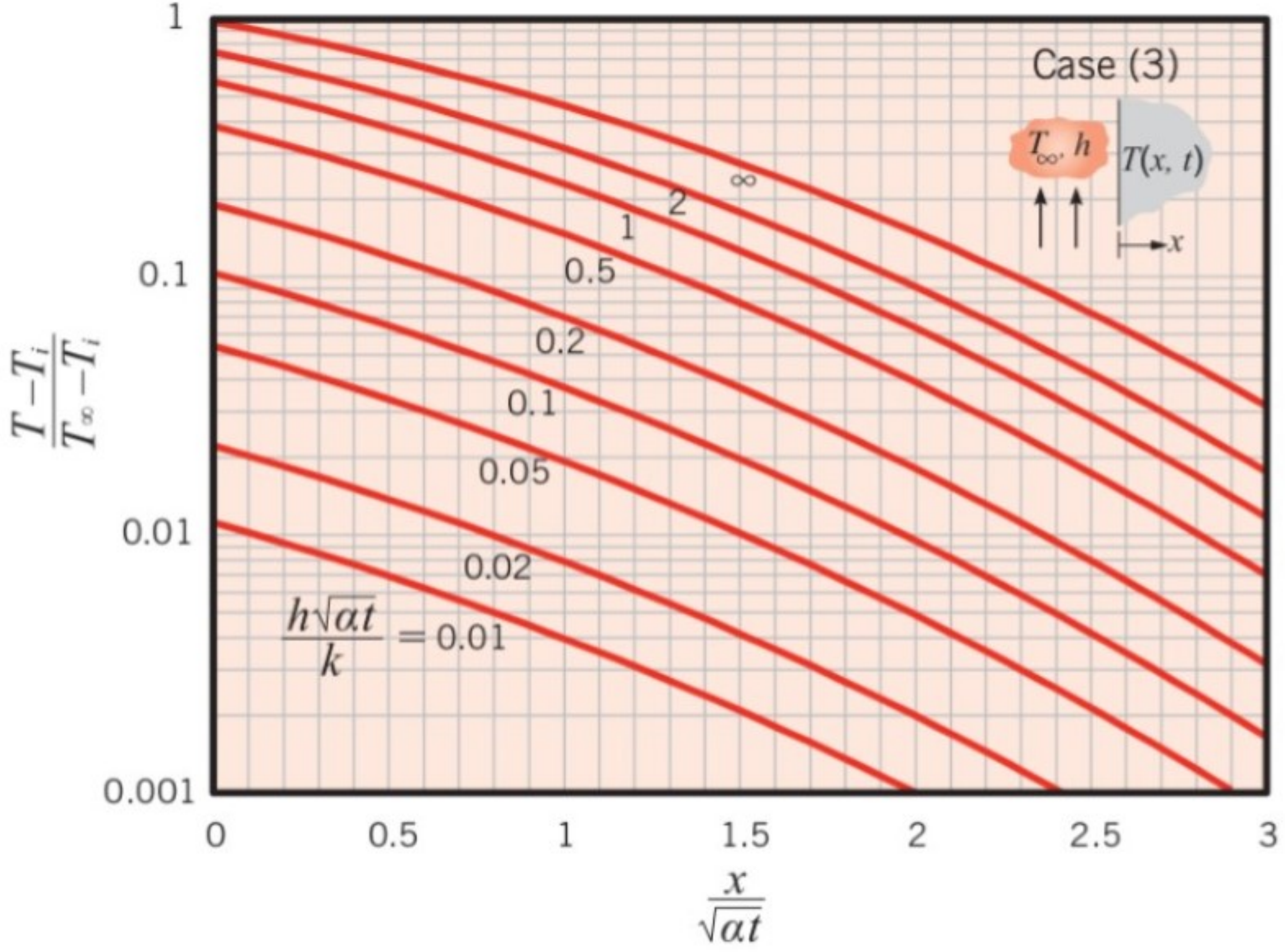
$$\operatorname{erfc}(w) = 1 - \operatorname{erf}(w)$$

Surface Convection

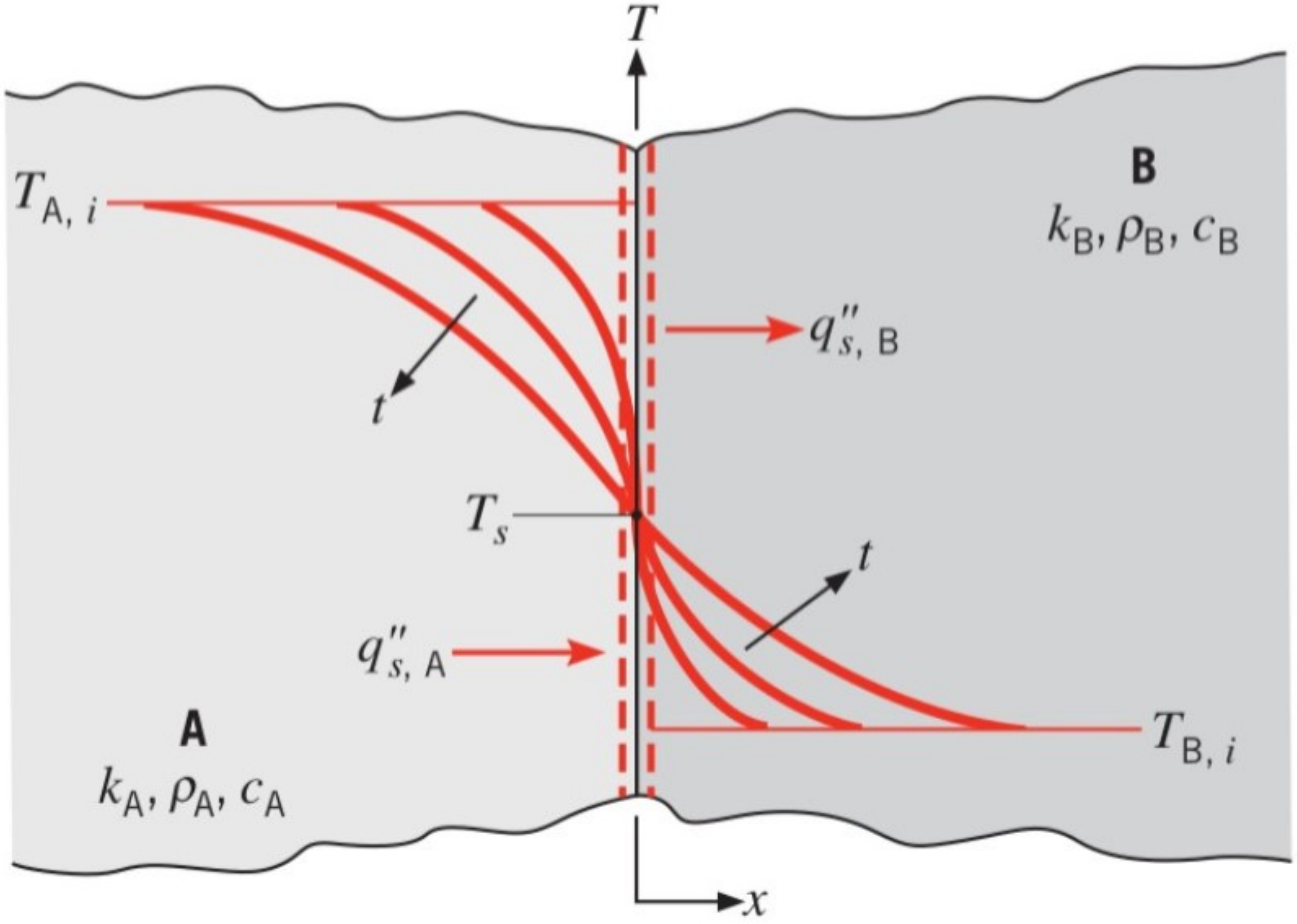
$$-k\frac{\partial T}{\partial x}\Big|_{x=0} = h(T_\infty - T(0,t))$$

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

Fig 5.8



Case (3)
T_∞, h
$T(x, t)$
x
∞
2
1
0.5
0.2
0.1
0.05
0.02
$\frac{h\sqrt{\alpha t}}{k} = 0.01$
1
0.1
0.01
0.001
$\frac{T - T_i}{T_\infty - T_i}$
0
0.5
1
1.5
2
2.5
3
$\frac{x}{\sqrt{\alpha t}}$


T
B
k_B, ρ_B, c_B
$T_{A,i}$
$q''_{s,B}$
t
T_s
t
$q''_{s,A}$
A
k_A, ρ_A, c_A
$T_{B,i}$
x

$$q''_{sA} = q''_{sB}$$

$$\frac{-k_A (T_s - T_{Ai})}{\sqrt{\pi \alpha_A t}} = \frac{k_B (T_s - T_{Bi})}{\sqrt{\pi \alpha_B t}}$$

$$T_s = \frac{\sqrt{k_A \rho_A c_A}\, T_{Ai} + \sqrt{k_B \rho_B c_B}\, T_{Bi}}{\sqrt{k_A \rho_A c_A} + \sqrt{k_B \rho_B c_B}}$$