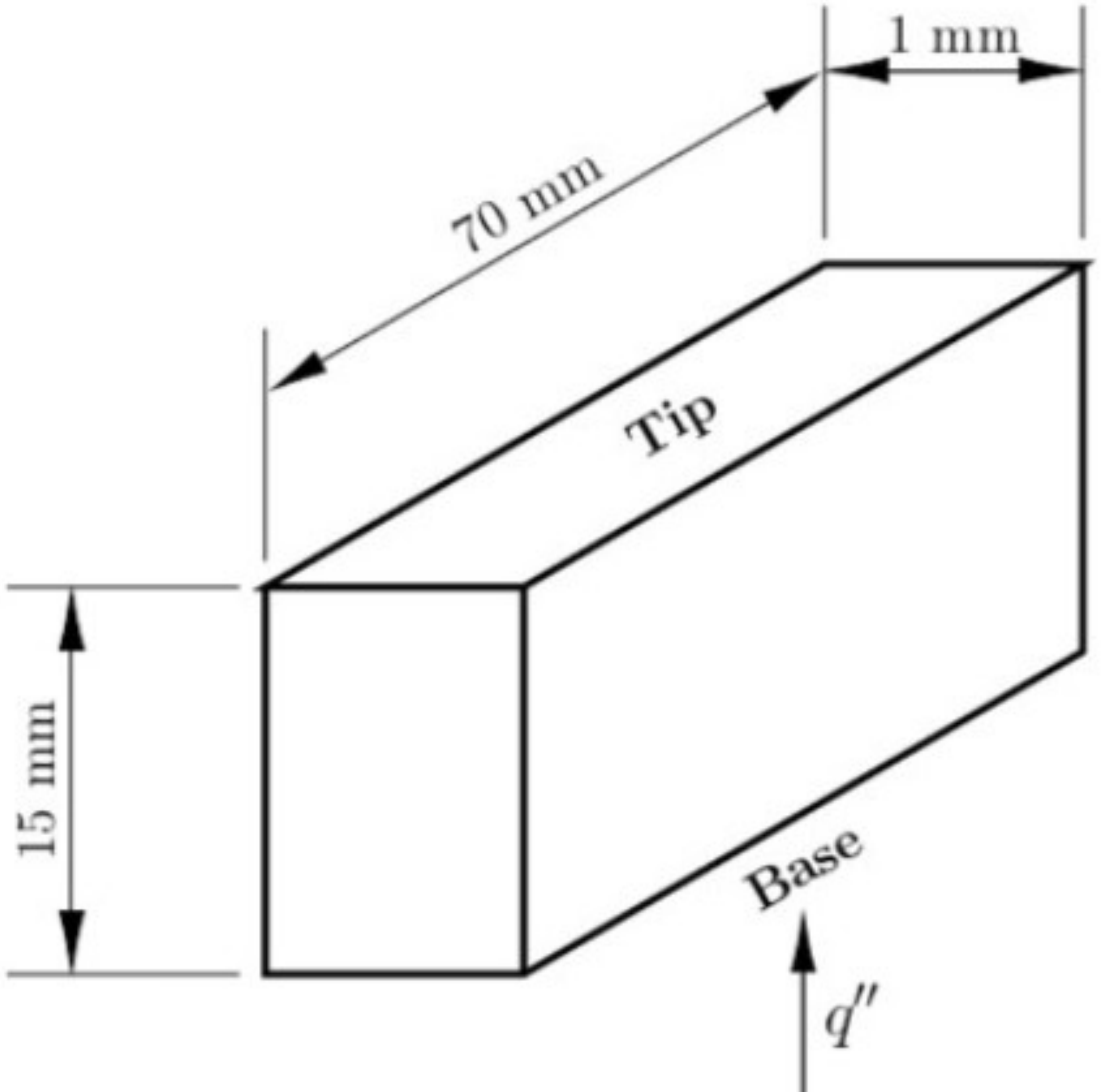
(30 points) Dr. Devine is using his computer to solve for the steady state temperature distribution using a finite difference method. His computer processor is producing a heat flux of $q'' = 1.36 \times 10^4 \ \frac{\mathrm{W}}{\mathrm{m}^2}$. One of the fins of the aluminum heatsink on the processor is shown below. Air blows over 5 sides of the fin (all sides except for the base) at a temperature, $T_\infty = 21\ °C$, and with a convection coefficient, $h = 50\ \frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{K}}$. Assuming steady state,

(a) what is the temperature at the base of the fin, and

(b) what is the temperature at the tip of the fin?



$$q_f = M \frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)}$$

$L = 0.015 \text{ m}$

$A_c = 0.07 \cdot 0.001 = 7\times10^{-5} \text{ m}^2$

$P = 2(0.07 + 0.001) = 0.142 \text{ m}$

$$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{50 \cdot 0.142}{237 \cdot 7\times10^{-5}}} = 20.7 \ \frac{1}{m}$$

$q_f = q'' A_c$

$M = 3.08 \text{ W}$

$$M = \sqrt{hPkA_c}\,\theta_b$$

$$\theta_b = 8.96\ °C$$

$$\theta_b = T_b - T_\infty$$

$$T_b = \theta_b + T_\infty = 8.96 + 21 = \boxed{30\ °C}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh(m(L-x)) + (h/mk)\sinh(m(L-x))}{\cosh(mL) + (h/mk)\sinh(mL)}$$

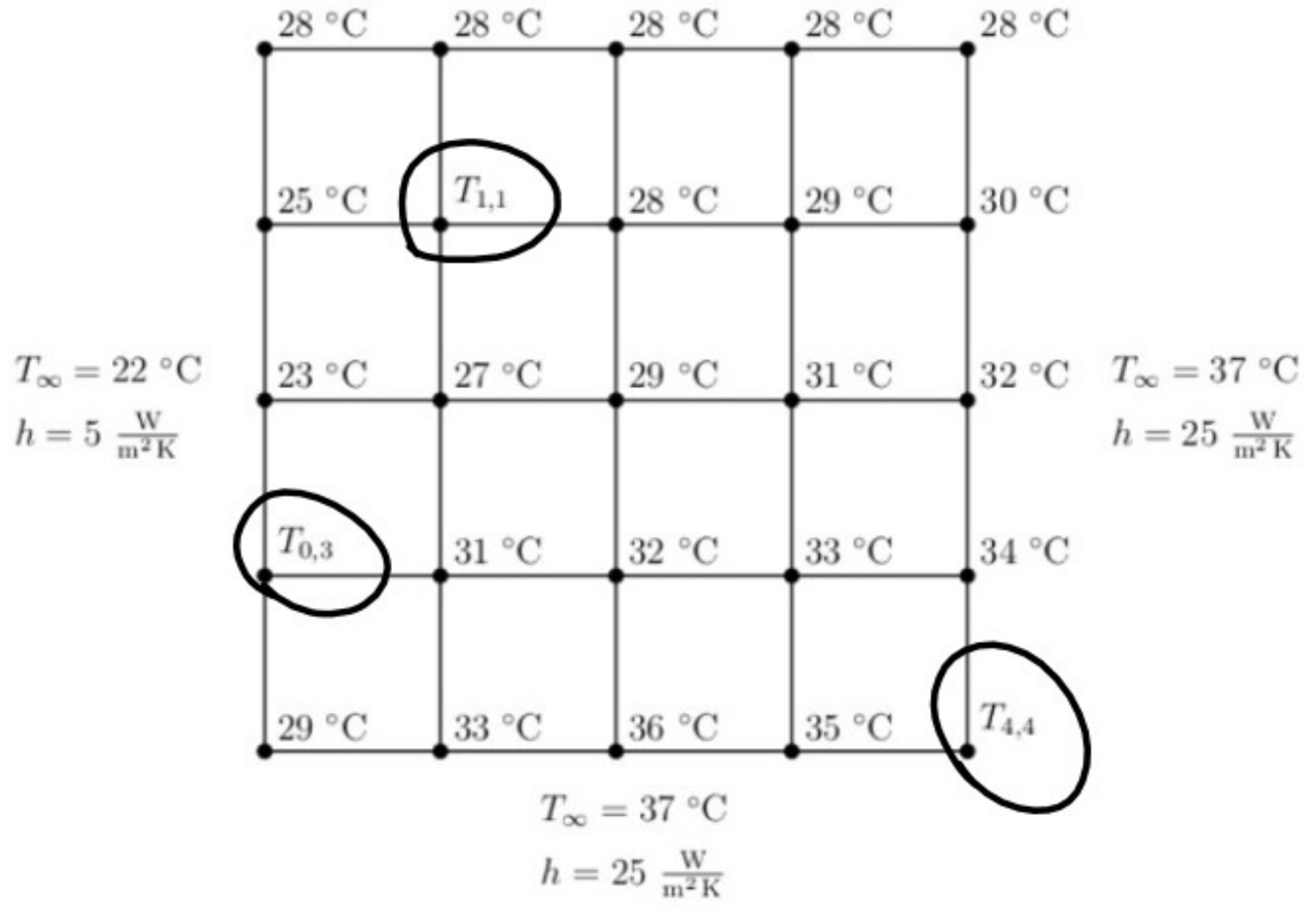
$$x = L$$

$$\frac{\theta}{\theta_b} = \frac{1}{\cosh(mL) + (h/mk)\sinh(mL)} = 0.95$$

$$\frac{T - T_\infty}{\theta_b} = 0.95$$

$$T = 0.95\theta_b + T_\infty = \boxed{29.5\ °C}$$

(15 points) After solving the temperature distribution from Problem 1, Dr. Devine's computer overheated. This caused it to lose the temperature at several nodes. Calculate the missing values. The material being simulated is oak hardwood, and the spacing of the grid is $\Delta x = 0.1$ m.



$$T_{0,1} + T_{2,1} + T_{1,0} + T_{2,1} - 4T_{1,1} = 0$$

$$T_{1,1} = \frac{T_{0,1} + T_{2,1} + T_{1,0} + T_{2,1}}{4} = \frac{28 + 23 + 27 + 25}{4} = \boxed{27°C}$$

$$T_{0,3} = \frac{2T_{1,3} + T_{0,2} + T_{0,4} + \frac{2h\Delta x}{k} T_\infty}{2\left(\frac{h\Delta x}{k} + 2\right)} = \frac{2 \cdot 31 + 23 + 29 + \frac{2 \cdot 5 \cdot 0.1}{0.16} 22}{2\left(\frac{5 \cdot 0.1}{0.16} + 2\right)} = \boxed{24.54\ °C}$$

$$T_{4,4} = \frac{T_{4,3} + T_{3,4} + 2\frac{h\Delta x}{k}T_{\infty}}{2\left(\frac{h\Delta x}{k}+1\right)} = \frac{35+34+2\frac{25\cdot 0.1}{0.16}37}{2\left(\frac{25\cdot 0.1}{0.16}+1\right)} = \boxed{36.85 \ {}^{\circ}C}$$

(30 points) With all the computer problems he is having, Dr. Devine is feeling hungry. He decides to get some food. He looks inside his (infinitely large) pantry and finds an (infinitely large) potato. This potato is strangely shaped, it is infinitely wide, infinitely deep, and is 60 mm thick. Dr. Devine places the potato in his (infinitely large) air fryer and sets it to 175 °C and waits for the minimum temperature in the potato to reach 100 °C. The potato is initially at 21 °C, the air fryer has a convection coefficient of 55 $\frac{\mathrm{W}}{\mathrm{m^2\,K}}$, and further details about the thermal properties of potatoes can be found in Appendix A.

(a) How long will Dr. Devine have to wait until his potato is cooked?

(b) Once the potato is cooked, what will the surface temperature of the potato be?

$x^* = 0$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = \frac{100 - 175}{21 - 175} = 0.487$$

$$\theta^* = C_1 \exp(-\zeta_1^2 \, Fo)$$

$$Bi = \frac{hL}{k} = \frac{55 \cdot 0.03}{0.56} = 2.98$$

$\zeta_1 = 1.19 \qquad C_1 = 1.21$

$$\theta^* = C_1 \exp(-\zeta_1^2 F_o)$$

$$\ln\left(\frac{\theta^*}{C_1}\right) = -\zeta_1^2 F_o \qquad \frac{-1}{\zeta_1^2} \ln\left(\frac{\theta^*}{C_1}\right) = F_o = 0.64$$

$$F_o = \frac{\alpha t}{L^2} \qquad t = \frac{F_o L^2}{\alpha} = 3831 \text{ s} = \boxed{64 \text{ min}} \qquad \alpha = \frac{k}{\rho c_p} = 1.51 \times 10^{-7} \frac{m^2}{s}$$

$$\theta^* = C_1 \exp(-\zeta_1^2 \ Fo) \cos(\zeta_1 x^*) \qquad x^* = \frac{x}{L} = \frac{L}{L} = 1$$

$$= 0.18$$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} \qquad T = \theta^* (T_i - T_\infty) + T_\infty = \boxed{147 \ ^\circ C}$$

(15 points) Annoyed at how long it is taking for his potato to cook, Dr. Devine looks in his pantry again and finds a more normal potato and places it in the microwave. The microwave can be considered to cause internal heat generation inside the potato, $\dot{q} = 8 \times 10^6 \ \frac{\text{W}}{\text{m}^3}$. If the potato is spherical with a radius of 35 mm, and there is no convection between the potato and the air in the microwave, how long will it take to cook this potato from the same initial temperature to the temperature listed in Problem 3?

$$3430 \frac{\cancel{J}\ \cancel{W}s}{\cancel{kg}\ \cancel{K}} \, 79 \cancel{K} \, \frac{1}{8\times10^6} \frac{\cancel{m^3}}{\cancel{W}} \, 1031 \frac{\cancel{kg}}{\cancel{m^3}} = \boxed{36.6 \text{ s}}$$

$$\Delta T = 100 - 21 = 79 \text{ K} \qquad 1 \text{ J} = 1 \text{ W s}$$

(10 points) By now Dr. Devine is very hungry, but his potato from Problem 4 is too hot to eat. He decides to submerge his potato 50 mm deep in a very large tub of sour cream straight from the fridge at 5 °C. What is the rate of heat transfer from the potato to the sour cream?

$$S = \frac{2\pi D}{1 - \frac{D}{4z}} = \frac{2\pi\ 0.07}{1 - \frac{0.07}{4 \cdot 0.05}} = 0.67\ \text{m}$$

$$q = Sk(T_1 - T_2) = 0.67 \cdot 0.56\,(100 - 5) = \boxed{36\ \text{W}}$$