

# Equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{dp_{\infty}}{dx} + v \frac{\partial^2 u}{\partial y^2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p} \left( \frac{\partial u}{\partial y} \right)^2$$

advection

y axis  
conduction

viscous  
dissipation

Conservation

of  
Mass

Kinematic  
viscosity  
 $\nu$

Momentum

Energy

Velocity  
 $v$

# Normalized Boundary Layers

$$x^* = \frac{x}{L}$$

$$y^* = \frac{y}{L}$$

$L$  characteristic length (flat plate)

$$u^* = \frac{u}{V}$$

$$v^* = \frac{v}{V}$$

$V$  free stream velocity

$$\bar{T}^* = \frac{T - T_s}{\bar{T}_\infty - \bar{T}_s}$$

**TABLE 6.1** The boundary layer equations and their  $y$ -direction boundary conditions in nondimensional form

Boundary Layer	Conservation Equation	Boundary Conditions		Similarity Parameter(s)
		Wall	Free Stream	
Velocity	$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = - \frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} \quad (6.35)$	$u^*(x^*, 0) = 0$	$u^*(x^*, \infty) = \frac{U_\infty(x^*)}{V} \quad (6.38)$	$Re_L = \frac{VL}{v} \quad (6.41)$
Thermal	$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}} \quad (6.36)$	$T^*(x^*, 0) = 0$	$T^*(x^*, \infty) = 1 \quad (6.39)$	$Re_L, Pr = \frac{v}{\alpha} \quad (6.42)$
Concentration	$u^* \frac{\partial C_A^*}{\partial x^*} + v^* \frac{\partial C_A^*}{\partial y^*} = \frac{1}{Re_L Sc} \frac{\partial^2 C_A^*}{\partial y^{*2}} \quad (6.37)$	$C_A^*(x^*, 0) = 0$	$C_A^*(x^*, \infty) = 1 \quad (6.40)$	$Re_L, Sc = \frac{v}{D_{AB}} \quad (6.43)$

$$Pr = \frac{c_p \mu}{k} = \frac{\gamma}{\alpha}$$

Prandtl number

Convection Similarity Parameters  $Re_L$ ,  $Pr$ , and  $Sc$

Nusselt Number

$$Nu = \frac{hL}{k_f} = \left. \frac{\partial T^*}{\partial y^*} \right|_{y^*=0}$$

$$\overline{Nu} = \frac{\bar{h} L}{k_f}$$

$$Re_L = \frac{F_I}{F_s} = \frac{\rho V^2 / L}{\mu V / L^2} = \frac{\rho V L}{\mu}$$

$F_I$  inertia forces

$F_s$  viscous forces

$$Pr = \frac{V}{\alpha}$$

$$\frac{\delta}{\delta_t} \approx Pr^n \quad n > 0$$

$n$  usually approximately  $\frac{1}{3}$

gasses       $Pr \approx 1$        $\delta_t \approx \delta$

liquid metal       $Pr \ll 1$        $\delta_t \gg \delta$

oils       $Pr \gg 1$        $\delta_t \ll \delta$