

# Turbulent Flow

$$C_{fx} = 0.0512 Re_x^{-\frac{1}{5}}$$

$$Re_{x,c} < Re_x < 10^8$$

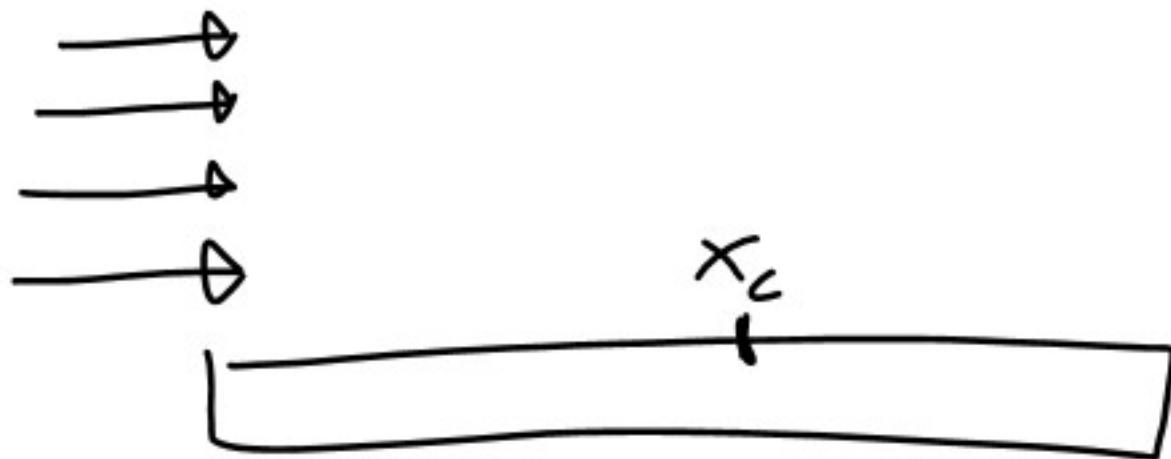
$$\delta = 0.37 \times Re_x^{-\frac{1}{5}}$$

$$Nu_x = St \cdot Re_x \cdot Pr = 0.0296 Re_x^{4/5} Pr^{1/3} \quad 0.6 < Pr < 60$$

Stanton Number

$$St = \frac{h}{\rho IV c_p} = \frac{Nu}{Re \cdot Pr}$$

Mixed Boundary Layer



$$\overline{h}_L = \frac{1}{L} \left( \int_0^{x_c} h_{\text{laminar}} dx + \int_{x_c}^L h_{\text{turbulent}} dx \right)$$

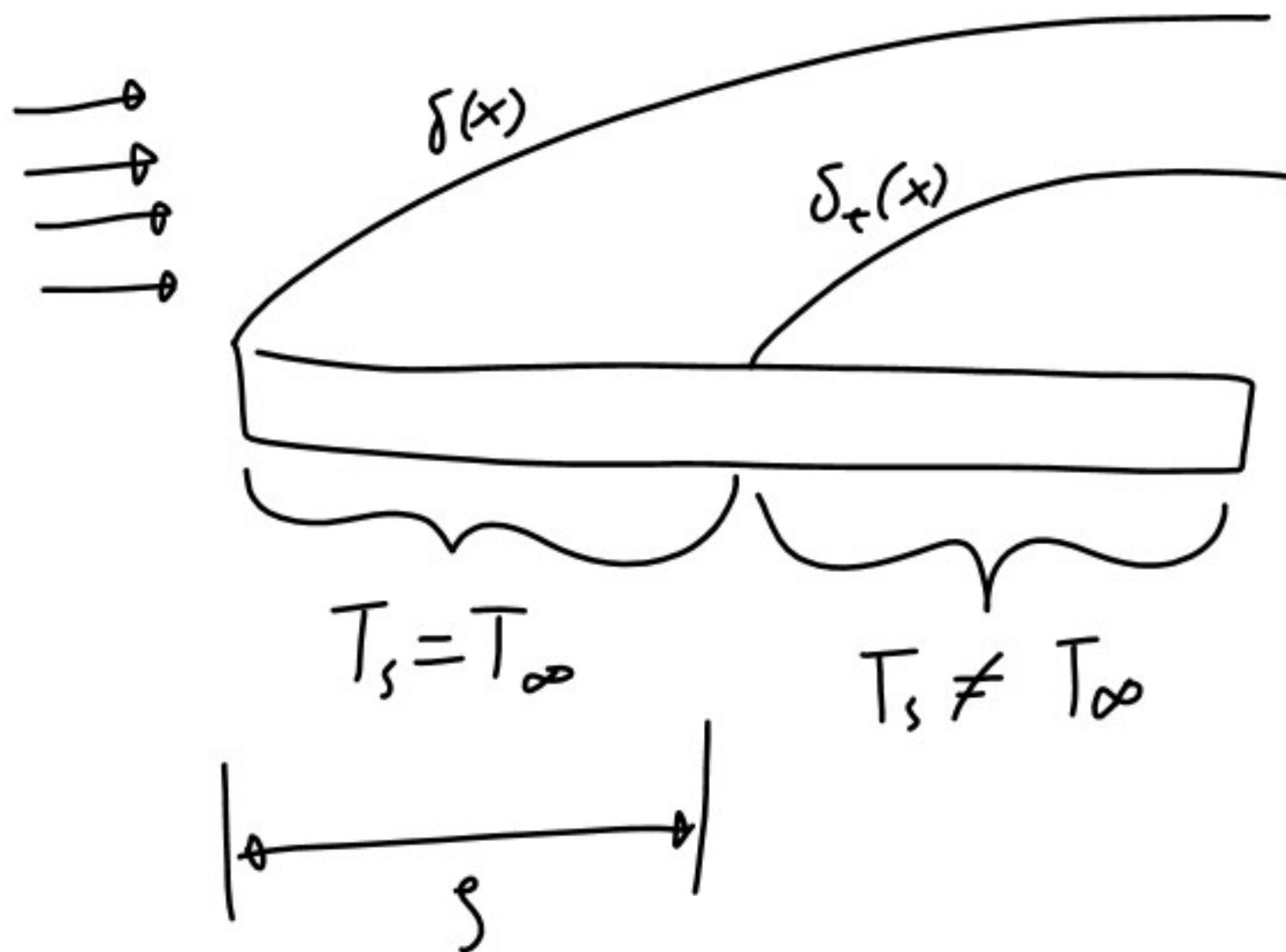
$$\overline{h}_L = \frac{k}{L} \left( 0.332 \left( \frac{U_\infty}{\nu} \right)^{1/2} \int_0^{x_c} \frac{dx}{x^{1/2}} + 0.0296 \left( \frac{U_\infty}{\nu} \right)^{2/5} \int_{x_c}^L \frac{dx}{x^{2/5}} \right) Pr^{1/3}$$

$$\boxed{\overline{N_u}_L = (0.037 Re_L^{2/5} - A) Pr^{1/3}}$$

$0.6 < Pr < 60$   
 $Re_{x_c} < Re_L < 10^8$

$$A = 0.037 Re_{x_c}^{2/5} - 0.664 Re_{x_c}^{1/2}$$

Unheated Starting Length



Laminar Flow

$$Nu_x = \frac{Nu_x|_{S=0}}{(1 - (\frac{S}{x})^{3/4})^{1/3}}$$

Turbulent Flow

$$Nu_x = \frac{Nu_x|_{S=0}}{(1 - (\frac{S}{x})^{9/10})^{1/9}}$$

$$\overline{Nu_L} = \overline{Nu_L} \Big|_{\beta=0} \cdot \frac{L}{L-\beta} \left(1 - \left(\frac{\beta}{L}\right)^{\frac{p+1}{p+2}}\right)^{\frac{p}{p+1}}$$

Laminar  $p=2$

Fully turbulent  $p=8$

$\beta > x_c$

7.2 Engine oil at 100°C and a velocity of 0.1 m/s flows over both surfaces of a 1-m-long flat plate maintained at 20°C. Determine:

- The velocity and thermal boundary layer thicknesses at the trailing edge.
- The local heat flux and surface shear stress at the trailing edge.
- The total drag force and rate of heat transfer per unit width of the plate.
- Plot the boundary layer thicknesses and local values of the surface shear stress, convection coefficient, and heat flux as a function of  $x$  for  $0 \leq x \leq 1$  m.

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$x = L = 1 \text{ m}$$

$$Re_x = \frac{\rho V L}{\mu}$$

$$T_f = \frac{T_s + T_\infty}{2} = \frac{20 + 100}{2} = 60^\circ \text{C}$$

$$= 273 + 60 = 333 \text{ K}$$

$$\rho = 865.3 \frac{\text{kg}}{\text{m}^3}$$

$$\mu \cdot 10^2 = 8.36 \frac{\text{Ns}}{\text{m}^2} \Rightarrow \mu = 8.36 \times 10^{-2} \frac{\text{Ns}}{\text{m}^2}$$

$$\frac{\delta}{\delta_t} = Pr^{\frac{1}{3}}$$

$$Nu_L = \frac{h_L L}{\kappa} = 0.332 Re_L^{1/2} Pr^{1/3}$$

$$\frac{\delta}{Pr^{\frac{1}{3}}} = \delta_t$$

$$Pr = 1205$$