intro.block Feedback control system block diagrams

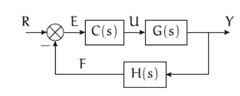


Figure block.1: a block diagram for a controller $C(s)$.

1 As we have already seen, a useful tool for designing control systems is the block diagram. The plant and the controller are represented as blocks. Usually a transfer function (or transfer function matrix) can describe the function of each block. A typical block diagram is shown in Fig. def.1.

2 In this configuration, a command function $R(s)$ is provided to the control system. The feedback $H(s)Y(s)$ is subtracted from $R(s)$ to give the error $E(s)$. This is fed to the controller $C(s)$. The output of the controller is the control effort $U(s)$, which is the input of the plant $G(s)$. The output $Y(s)$, after being fed back as $H(s)Y(s)$, is what the control system is attempting to make equal to the command $R(s)$, therefore, ideally $E(s) = 0$.

3 Block diagrams express algebraic relationships. (The blocks do not dynamically "load" each other.) In the case of Fig. def.2, the relationships are

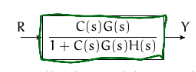


Figure block.2: a block diagram with the corresponding closed-loop transfer function block, derived in Example intro.block-1.

$$E(s) = R(s) - F(s) \tag{1a}$$
$$U(s) = C(s)E(s) \tag{1b}$$
$$Y(s) = G(s)U(s) \tag{1c}$$
$$F(s) = H(s)Y(s). \tag{1d}$$

The closed-loop tranfer function is defined as $Y(s)/R(s)$. This important transfer function shows how the system should respond to commands, of key importance for most performance criteria.

Example intro.block−1 **re: Closed−loop transfer function**

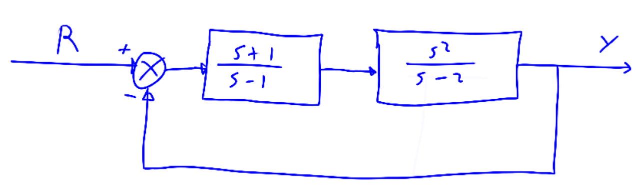
Given the feedback block diagram of Fig. def.1 (left), solve for the closed loop transfer function $Y(s)/R(s)$.

$$\begin{aligned}
Y(s) &= G(s)U(s) \\
&= G(s)C(s)E(s) \\
&= G(s)C(s)(R(s) - F(s)) \\
&= G(s)C(s)(R(s) - H(s)Y(s)) \\
&= G(s)C(s)R(s) - G(s)C(s)H(s)Y(s)
\end{aligned}$$

$$Y(s) + G(s)C(s)H(s)Y(s) = G(s)C(s)R(s)$$

$$Y(s)(1 + G(s)C(s)H(s)) = G(s)C(s)R(s)$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G(s)C(s)}{1 + G(s)C(s)H(s)}}$$



$$H(s) = 1$$
$$C(s) = \frac{s+1}{s-1}$$
$$G(s) = \frac{s^2}{s-2}$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)C(s)}{1 + G(s)C(s)H(s)} = \frac{\frac{s^2}{s-2}\frac{s+1}{s-1}}{1 + \frac{s^2}{s-2}\frac{s+1}{s-1}\cancel{C(s)}} \quad \frac{s-2}{s-2}$$

$$= \frac{s^2\frac{s+1}{s-1}}{(s-2) + s^2\frac{s+1}{s-1}} \quad \frac{s-1}{s-1}$$

$$= \frac{s^2(s+1)}{(s-2)(s-1) + s^2(s+1)}$$

$$= \frac{s^3 + s^2}{s^2 - 2s - s + 2 + s^3 + s^2}$$

$$= \boxed{\frac{s^3 + s^2}{s^3 + 2s^2 - 3s + 2}}$$