trans.exe Exercises for Chapter trans

Exercise trans.apiarian

A control system has dominant closed-loop poles at $-8 \pm j3$. Under the second-order assumption, what is its settling time?

$\omega_n = \sqrt{(-8)^2 + 3^2} = \sqrt{64 + 9} = \sqrt{73} = 8.54$

$-\zeta\omega_n = -8 \Rightarrow \zeta = \frac{8}{\omega_n} = \frac{8}{8.54} = 0.94$

$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.94(8.54)} = \boxed{0.5\ s}$

Exercise trans.pericentral

Consider the block diagram of Fig. exe.1. Let the plant G have transfer function

$$G(s) = \frac{9}{(s+4)(s^2+3s+9)}, \tag{1}$$

the feedback transfer function $H(s) = 1$, and the controller C have transfer function

$$C(s) = K \tag{2}$$

where $K \in \mathbb{R}$ is some gain.

1. Determine the closed loop transfer function $Y(s)/R(s)$. ✓
2. For $K = 4$ and a unit step input to the closed-loop system, what are the second-order approximations of the peak time T_p, rise time T_r, settling time T_s, and percent overshoot $\%OS$? ✓
3. For $K = 4$ and a unit step input to the closed-loop system, simulate to estimate peak time T_p, rise time T_r, settling time T_s, and percent overshoot $\%OS$.
4. Compare the second-order approximations with the simulated results. Explain the differences and similarities.

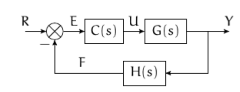

Figure exe.1: a block diagram with a controller $C(s)$.

$$\frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

$$= \frac{K\frac{9}{(s+4)(s^2+3s+9)}}{1 + K\frac{9}{(s+4)(s^2+3s+9)}} \frac{(s+4)(s^2+3s+9)}{(s+4)(s^2+3s+9)}$$

$$= \frac{9K}{(s+4)(s^2+3s+9) + 9K} = \frac{9K}{s^3+3s^2+9s+4s^2+12s+36+9K}$$

$$= \boxed{\frac{9K}{s^3+7s^2+21s+36+9K}}$$

Poles $-0.72 \pm 3.53i$

$\sqrt{0.72^2 + 3.53^2} = \omega_n = 3.6$

$-0.72 = -\zeta\omega_n$ $\quad \frac{0.72}{3.6} = \zeta = 0.2$

$T_r\omega_n = 1.2$ $\quad \boxed{T_r = \frac{1.2}{3.6} = 0.33\ s}$ from sim $0.4\ s$

$\omega_d = 3.53$

$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.53} = \boxed{0.89 = T_p}$ from sim $1.1\ s$

$\%OS = 100 \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) = 100 \exp\left(\frac{-0.2\pi}{\sqrt{1-0.2^2}}\right)$

$= 100 \exp(-0.641)$

$\boxed{\%OS = 52\ \%}$ from sim 43%

$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.2(3.6)}$

$\boxed{T_s = 5.56\ s}$ from sim $4.9\ s$

steady

Steady–state response performance

After the transient response has settled—that is, reached steady-state—the system may or may not be in a desirable state. If the response asymptotically approaches any state other than that commanded, it is said to have steady-state error. These arise from three primary sources:

1. nonlinearities, like backlash in gears—we won't explore this one;
2. disturbances, like those from the environment; and
3. input (command) type and the plant dynamics.

We will focus our attention on item 3; item 2 is similar.