

Nisc 4.23

for each pair of 2nd order system specifications find the location of the pair of poles

a) %OS = 12%  $T_s = 0.6$  s

b) %OS = 10%  $T_p = 5$  s

c)  $T_s = 7$  s  $T_p = 3$  s

$$a) \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = \frac{-\ln(0.12)}{\sqrt{\pi^2 + \ln^2(0.12)}} = 0.56$$

$$T_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = \frac{4}{\zeta T_s} = \frac{4}{0.56(0.6)} = 11.9$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 11.9 \sqrt{1 - 0.56^2} = 9.85$$

$$p = -\zeta \omega_n \pm \omega_d j = -0.56(11.9) \pm 9.85 j$$

$$= \boxed{-6.66 \pm 9.85 j}$$

c)  $T_p = \frac{\pi}{\omega_d}$   $\omega_d = \omega_n \sqrt{1 - \zeta^2}$   $T_s = \frac{4}{\zeta \omega_n}$

$$T_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$\omega_n = \frac{4}{\zeta T_s}$$

$$T_p = \frac{\pi}{\frac{4}{\zeta T_s} \sqrt{1 - \zeta^2}}$$

$$3 = \frac{\pi}{\frac{4}{\zeta 7} \sqrt{1 - \zeta^2}}$$

$$(3)^2 = \left( \frac{7\pi \zeta}{4 \sqrt{1 - \zeta^2}} \right)^2$$

$$9 = \frac{49 \pi^2 \zeta^2}{16 (1 - \zeta^2)}$$

$$9(16)(1 - \zeta^2) = 49 \pi \zeta^2$$

$$144 - 144 \zeta^2 = 49 \pi \zeta^2$$

$$144 = (49 \pi + 144) \zeta^2$$

$$\frac{144}{49 \pi + 144} = \zeta^2$$

$$\sqrt{\frac{144}{49 \pi + 144}} = \zeta = 0.43$$