freq.bodesketch Sketching Bode plots

1 We can use MATLAB's `bode` command to create Bode plots from LTI system models. However, we must understand how these plots relate to their transfer functions. In this section, we learn to sketch Bode plots in order to deepen our intuition of this relationship.

2 Let $H(s) = \prod_i H_i(s)$; that is, let $H(s)$ be the product of several factors $H_i(s)$. The magnitude and phase are

$$|H(s)| = \prod_i |H_i(s)| \quad \text{and} \quad \angle H(s) = \sum_i \angle H_i(s). \tag{1}$$

The Bode plot consists of plots of $20\log_{10}|H(s)|$ and $\angle H(s)$ with $s \mapsto j\omega$. The magnitude and phase expressions, become

$$20\log_{10}|H(j\omega)| = \sum_i 20\log_{10}|H_i(j\omega)| \quad \text{and} \quad \angle H(j\omega) = \sum_i \angle H_i(j\omega). \tag{2}$$

This result means we can graphically sum both the magnitude and phase Bode plots of the individual factors of $H(s)$, as long as we are adding magnitudes in dB.

Example freq.bodesketch–1 re: a transfer function under analysis

Given the transfer function

$$H(s) = \frac{200000(s+1)}{s^3 + 110s^2 + 11000s + 100000}$$

answer the following questions and imperatives.

a Sketch a Bode plot on Fig. bodesketch.1.

b Confirm the accuracy of the sketch in Matlab, using the functions `bode` and `tf`.

c If the input to a system with this transfer function is $5\sin(\omega t + \pi/7)$, what is the output amplitude and phase for
   i $\omega = 1$ rad/s,
   ii $\omega = 10$ rad/s, and
   iii $\omega = 1000$ rad/s?

   Use Matlab's function `evalfr` to perform the calculations.

a

To sketch the transfer function, we must decompose the transfer function into multiple simple factors. First, we can find the poles:

$$-10, -50 + j86.6, -50 - j86.6,$$

which tells us we have a complex conjugate pair and a single real pole. Factoring, accordingly,

$$\begin{aligned} H(s) &= 200000(s+1) \cdot \frac{1}{s+1} \cdot \frac{1}{s^2 + 100s + 10000} \\ &= 2(s+1) \cdot \frac{1}{s/10 + 1} \cdot \frac{100^2}{s^2 + 2 \cdot 0.5 \cdot 100s + 100^2}. \end{aligned}$$

The sketch is shown in Fig. bodesketch.1.

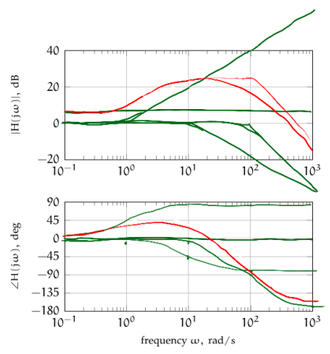

Figure bodesketch.1: a Bode plot for Example freq.bodesketch-1.

b

See the code listing below.

```
sys = 2e5*...
tf(...
  [1,1],...
  [1,110,11000,1e5]...
);
bode(sys);
```

c

The output amplitude is always $5|H(j\omega)|$ and output phase is always $\pi/7 + \angle H(j\omega)$. We could estimate them from the Bode plot sketch, but we instead choose to evaluate the Matlab transfer function, as in the listing below.

```
in_amp = 5;
in_phase = pi/7; % rad
omega_a = [1,10,1e3]; % rad/s
for i = 1:length(omega_a)
  H_eval = evalfr(sys,j*omega_a(i));
  H_mag = abs(H_eval);
  H_phase = angle(H_eval);
  out_amp = 5*H_mag;
  out_phase = in_phase + H_phase;
  sprintf(...
    ['For input angular freq %0.2g,\n',...
      ' input amplitude %0.2g,\n',...
      ' input phase %0.2g,\n',...
      ' H magnitude %0.2g, and\n',...
      ' H phase %0.2g,\n',...
      ' the output amplitude is %0.2g and\n',...
      ' the output phase is %0.2g.\n'...
    ],...
    omega_a(i),...
    in_amp,...
    in_phase,...
    H_mag,...
    H_phase,...
    out_amp,...
    out_phase...
  )
end
```

The output amplitudes are 14, 71, and 1 and the output phases are 1.1, 1, -2.6 rad.