freqd.gain Transient response design by adjusting the gain

The following design procedure allows us to design for a desired percent overshoot. A similar procedure could be followed to design for a desired damping ratio.

1. Generate open-loop Bode plots with some convenient initial gain K_i.
2. Use either Fig. freqtime.1 or Eq. 6 and Eq. 7 to find the desired phase margin Φ_M.
3. From the Bode phase plot, determine the frequency ω_{Φ_M} at which (180 deg minus the absolute value of) the phase is equal to the desired phase margin.
4. Change the gain to be such that the magnitude plot would intersect 0 dB at ω_{Φ_M}.

Example freqd.gain−1 re: Percent overshoot design by adjusting the gain

Design a unity feedback gain controller for a system with the plant

$$G(s) = \frac{10}{(s+90)(s+30)}$$

such that the percent overshoot %OS is approximately 20%.

$$\%OS = 20\%$$

$$\zeta = \frac{-\ln(20/100)}{\sqrt{\pi^2 + \ln^2(20/100)}} = \frac{1.6}{3.53} = 0.45$$

$$\Phi_M = \operatorname{atan}\left(\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}}\right) = \operatorname{atan}\left(\frac{0.9}{\sqrt{-0.4 + \sqrt{1.16}}}\right)$$

$$= \operatorname{atan}\left(\frac{0.9}{\sqrt{0.67}}\right)$$

$$= \operatorname{atan}(1.1) = 47.8^\circ$$

$$180 - 47.8 = 132^\circ$$

$$-65 \text{ dB} \Rightarrow \quad -65 = 20 \log_{10} \text{mag}$$

$$10^{\left(\frac{-65}{20}\right)} = \text{mag} = 5.62 \times 10^{-4}$$

$$\frac{1}{5.62 \times 10^{-4}} = 1778$$

$$\omega_{BW} \approx 200 \text{ rad/s}$$

$$\omega_{BW} = \frac{4}{T_s \zeta}\left((1-2\zeta^2) + (4\zeta^4 - 4\zeta^2 + 2)^{1/2}\right)^{1/2}$$

$$T_s = \frac{4}{\omega_{BW}\zeta}\left((1-2\zeta^2) + (4\zeta^4 - 4\zeta^2 + 2)^{1/2}\right)^{1/2}$$

$$T_s = \frac{4}{200(0.45)} 1.32 = 0.05 \text{ s}$$