Saving Plots for lab 1

```
robot.save_pdf();
robot.save_png();
robot.save_jpg();
```

```
while dx ~= 0
    :
    if robot.i == 25
    robot.save_pdf();
    end
end
```

step 25.pdf

Discrete Systems and Robot Motion

Motion Models

Lab 1

input ΔX

Example

wheeled robot

current controlled motors

$$\underbrace{\begin{bmatrix}\dot{V}\end{bmatrix}}_{\dot{X}} = \underbrace{\begin{bmatrix}\frac{-Br}{m}\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}V\end{bmatrix}}_{X} + \underbrace{\begin{bmatrix}\frac{K_m r}{m}\end{bmatrix}}_{B}\underbrace{\begin{bmatrix}I_s\end{bmatrix}}_{u}$$

V robot velocity

B wheel bearing damping

r wheel radius

m robot mass

K_m motor torque constant

I_s motor current

$$\dot{x} = V$$

$$\begin{bmatrix}\dot{V} \\ \dot{x}\end{bmatrix} = \begin{bmatrix}\frac{-Br}{m} & 0 \\ 1 & 0\end{bmatrix}\begin{bmatrix}V \\ x\end{bmatrix} + \begin{bmatrix}\frac{K_m r}{m} \\ 0\end{bmatrix}\begin{bmatrix}I_s\end{bmatrix}$$

$$\dot{\underline{X}} = A\underline{X} + Bu \qquad y = C\underline{X} + Du$$

$$\underline{X}_{t+1} = A_d\underline{X}_t + B_d u_t \qquad y_t = C_d\underline{X}_t + D_d u_t$$

Discretization

$$A_d = e^{A\Delta t}$$

$$B_d = A^{-1}(A_d - I)B$$

$$C_d = C$$

$$D_d = D$$

Approximation

$$A_d = I + \Delta t A$$

$$B_d = \Delta t B$$

Example

$$\begin{bmatrix} \dot{V} \\ \dot{X} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-Br}{m} & 0 \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} V \\ X \end{bmatrix} + \begin{bmatrix} \frac{K_m r}{m} \\ 0 \end{bmatrix} \begin{bmatrix} I_s \end{bmatrix}$$

$$X_t = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad u_t [1]$$

$$X_{t+1} = A_d X_t + B_d u_t$$

$$= \begin{bmatrix} 1 - \Delta t \frac{Br}{m} & 0 \\ \Delta t & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta t \frac{K_m r}{m} \\ 0 \end{bmatrix} [1] = \begin{bmatrix} \Delta t \frac{K_m r}{m} \\ 0 \end{bmatrix}$$

$$A_d = e^{A \Delta t}$$

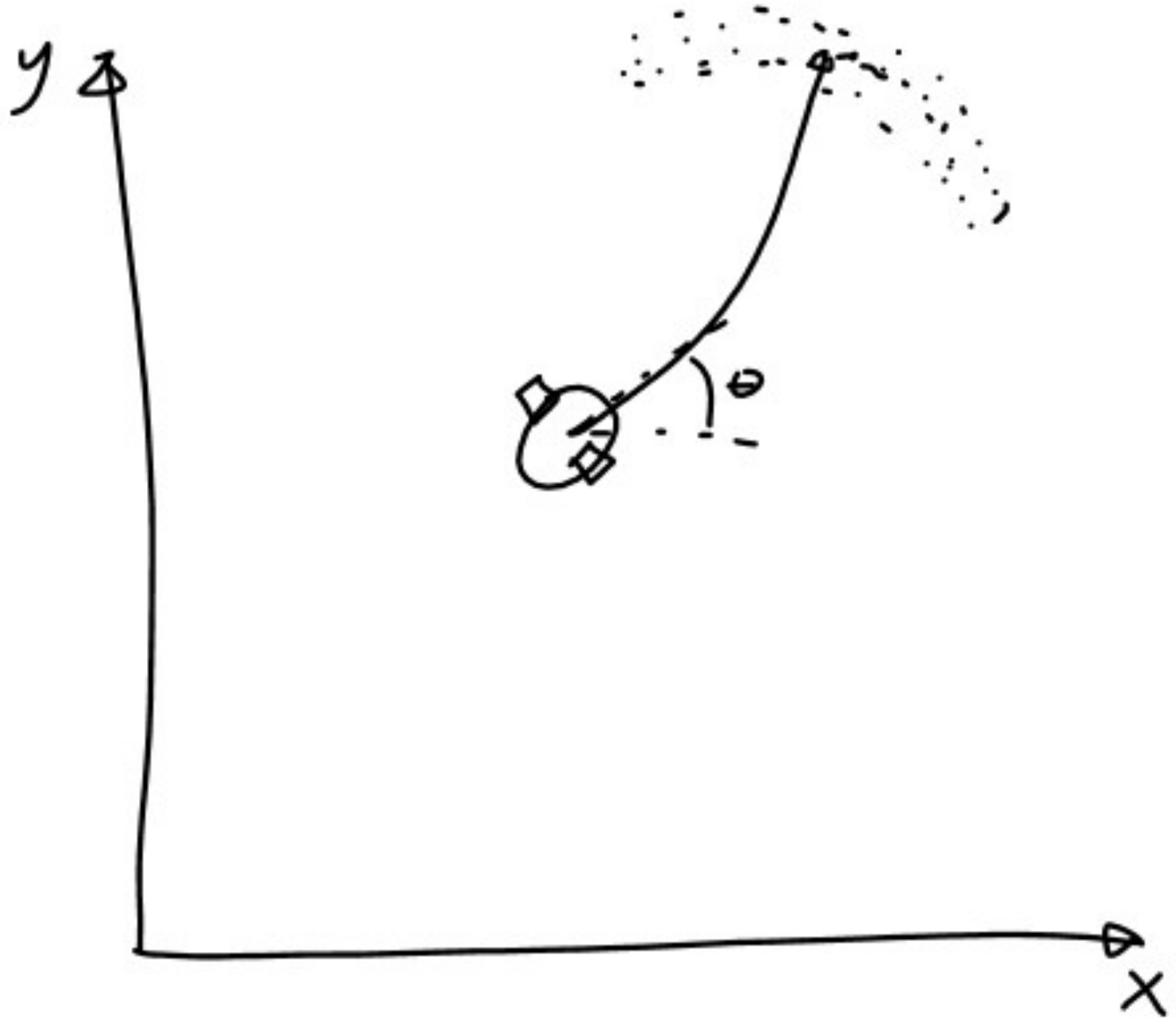
$$\det(A) = \left(\frac{-Br}{m}\right) 0 - 0(1) = 0$$

$$B_d = A^{-1}(A_d - I)B$$

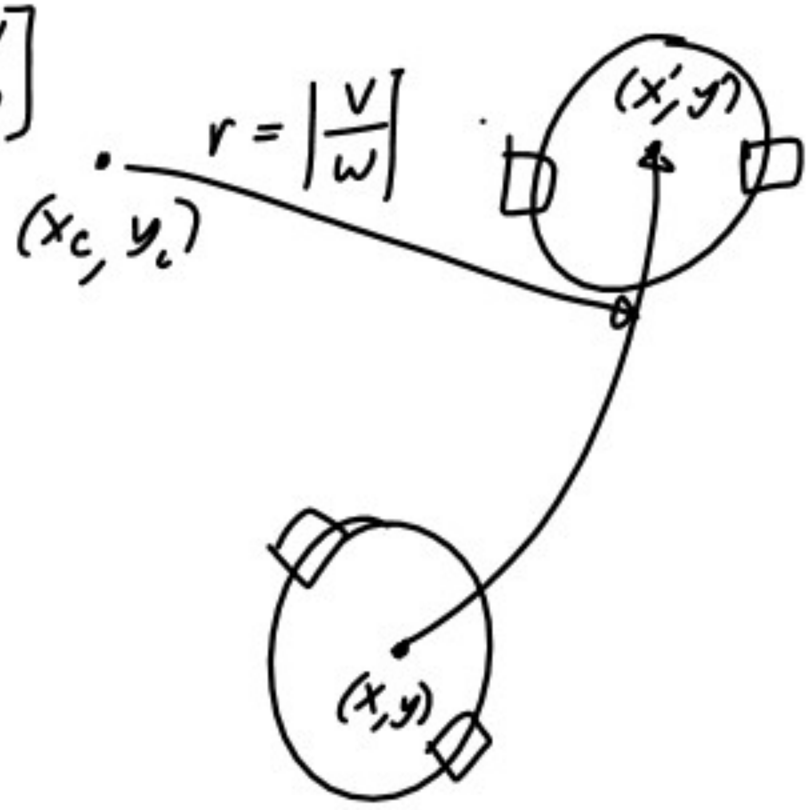
$$A_d = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \Delta t \begin{bmatrix} \frac{-Br}{m} & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 - \Delta t \frac{Br}{m} & 0 \\ \Delta t & 1 \end{bmatrix}$$

$$B_d = \Delta t \begin{bmatrix} \frac{K_m r}{m} \\ 0 \end{bmatrix} = \begin{bmatrix} \Delta t \frac{K_m r}{m} \\ 0 \end{bmatrix}$$

Robot Motion



$$\underline{X} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \qquad u = \begin{bmatrix} v \\ w \end{bmatrix}$$



$$x_c = x - \frac{v}{w}\sin\theta$$

$$y_c = y + \frac{v}{w}\cos\theta$$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x_c + \frac{v}{w}\sin(\theta + w\Delta t) \\ y_c - \frac{v}{w}\cos(\theta + w\Delta t) \\ \theta + w\Delta t \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} -\frac{v}{w}\sin\theta + \frac{v}{w}\sin(\theta + w\Delta t) \\ \frac{v}{w}\cos\theta - \frac{v}{w}\cos(\theta + w\Delta t) \\ w\Delta t \end{bmatrix}$$

Noise

$$\hat{u} = \begin{bmatrix} \hat{v} \\ \hat{\omega} \end{bmatrix} = \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} \varepsilon_{\alpha_1 v^2 + \alpha_2 \omega^2} \\ \varepsilon_{\alpha_3 v^2 + \alpha_4 \omega^2} \end{bmatrix}$$

ε_{b^2} value sampled from a PDF with zero mean and variance b^2

Gaussian Distribution

Triangular Distribution

Nonlinear Systems

$$\dot{\underline{X}} = A\underline{X} + Bu$$

$$\underline{X}_{t+1} = A_d \underline{X}_t + B_d u_t$$

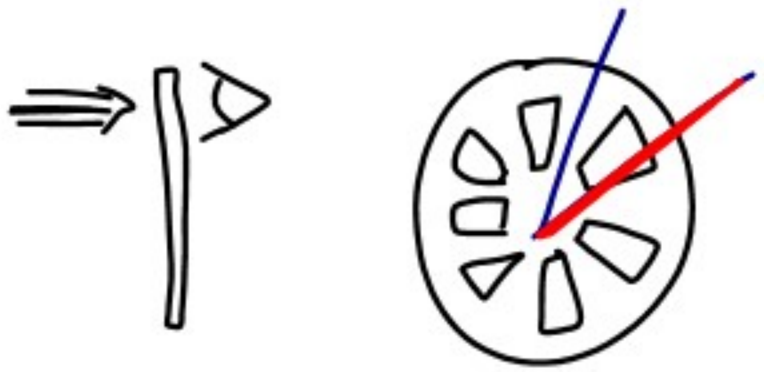
$$\dot{\underline{X}} = f(t, \underline{X}, u)$$

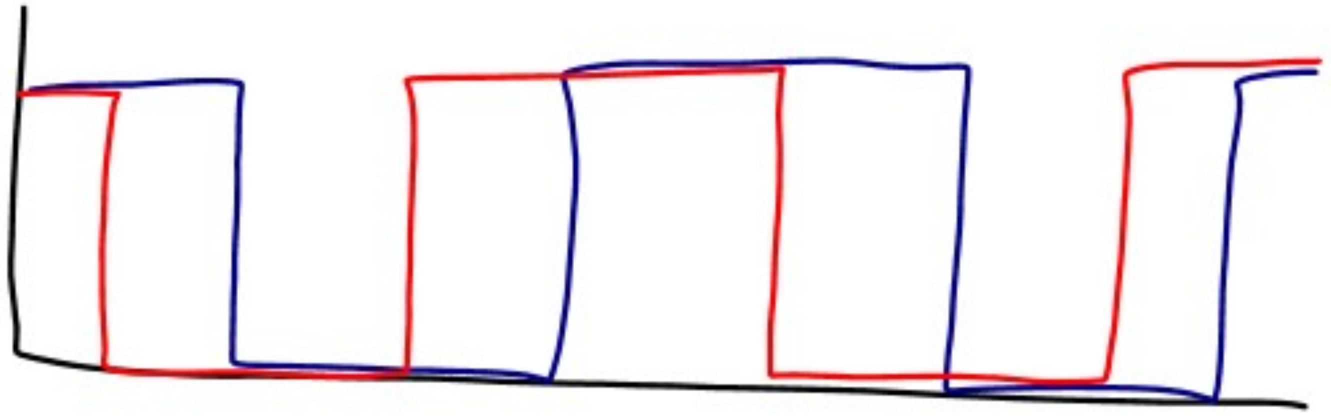
$$\underline{X}_{t+1} = g(t, \underline{X}_t, u_t)$$

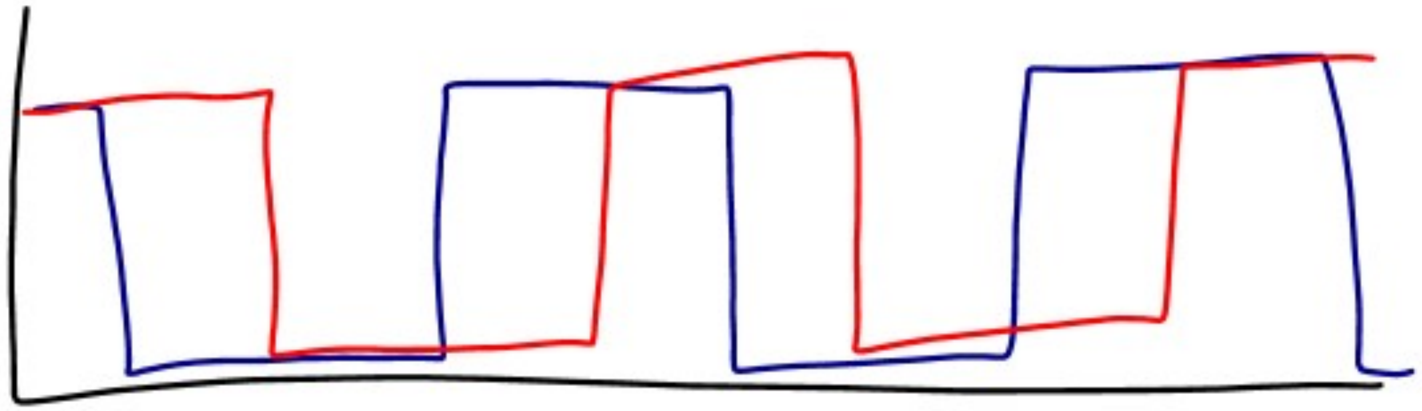
Odometry

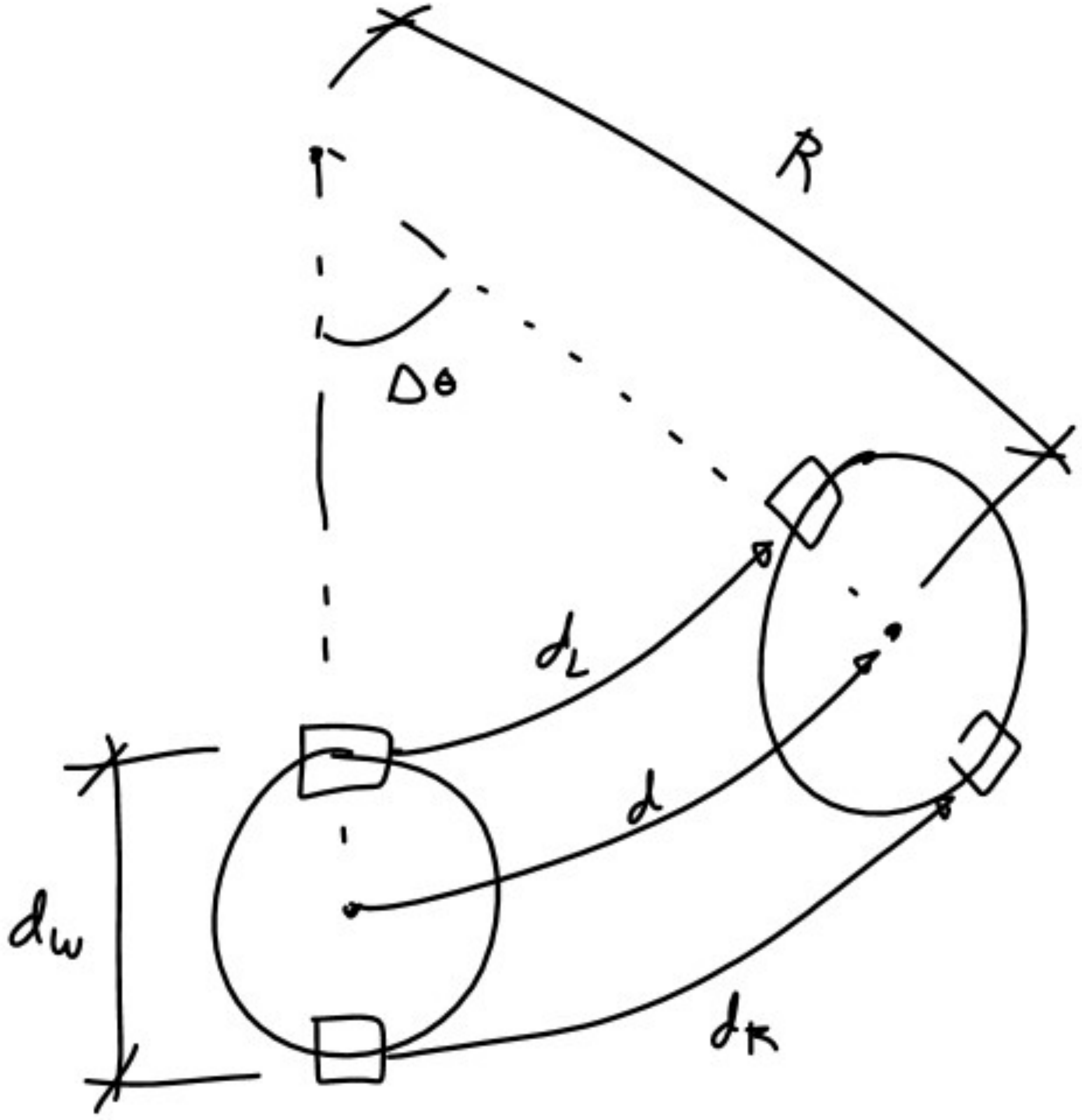
Measure Wheel Displacement

Quadrature Encoders









$$d = R\Delta\theta$$

$$d_L = \left(R - \frac{d_W}{2}\right)\Delta\theta$$

$$d_R = \left(R + \frac{d_W}{2}\right)\Delta\theta$$

$$\Delta\theta = \frac{d_R - d_L}{d_W}$$

$$R = d_L\left(\frac{d_W}{d_R - d_L}\right) + \frac{d_W}{2}$$

$$d = \frac{d_R + d_L}{2}$$

$$V = \frac{d}{\Delta t}$$

$$w = \frac{\Delta\theta}{\Delta t}$$

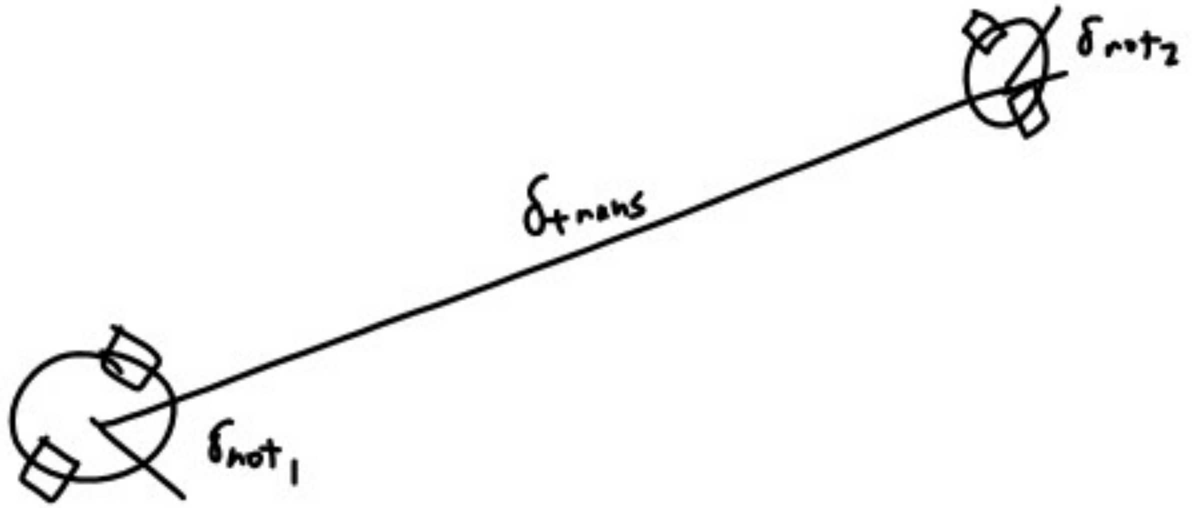
Some Robots Do Odometry Automatically

$$\bar{X}_t = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \bar{\theta} \end{bmatrix} \qquad \bar{X}_{t+1} = \begin{bmatrix} \bar{x}' \\ \bar{y}' \\ \bar{\theta}' \end{bmatrix}$$

$$\delta_{rot1} = atan2(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



$$\hat{\delta}_{rot1} = \delta_{rot1} + \varepsilon_{\alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2}$$

$$\hat{\delta}_{trans} = \delta_{trans} + \varepsilon_{\alpha_3 \delta_{trans}^2 + \alpha_4 \delta_{rot1}^2 + \alpha_4 \delta_{rot2}^2}$$

$$\hat{\delta}_{rot2} = \delta_{rot2} + \varepsilon_{\alpha_1 \delta_{rot2}^2 + \alpha_2 \delta_{trans}^2}$$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1}) \\ \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1}) \\ \hat{\delta}_{rot1} + \hat{\delta}_{rot2} \end{bmatrix}$$

Arctangent 2 Function

$\operatorname{atan}(y, x) = \operatorname{atan}\left(\frac{y}{x}\right)$

$\operatorname{atan}(1/0)$ $\quad$ $\operatorname{atan}(-1/0)$

$\uparrow$ $\quad$ $\downarrow$

$\operatorname{atan2}(1, 0)$ $\quad$ $\operatorname{atan2}(-1, 0)$