

Markov  
Localization

Kalman  
Filter

Extended  
Kalman  
Filter

Particle  
Filter

multimodal PDF

unimodal PDF

unimodal PDF

multimodal PDF

nonlinear

linear

nonlinear

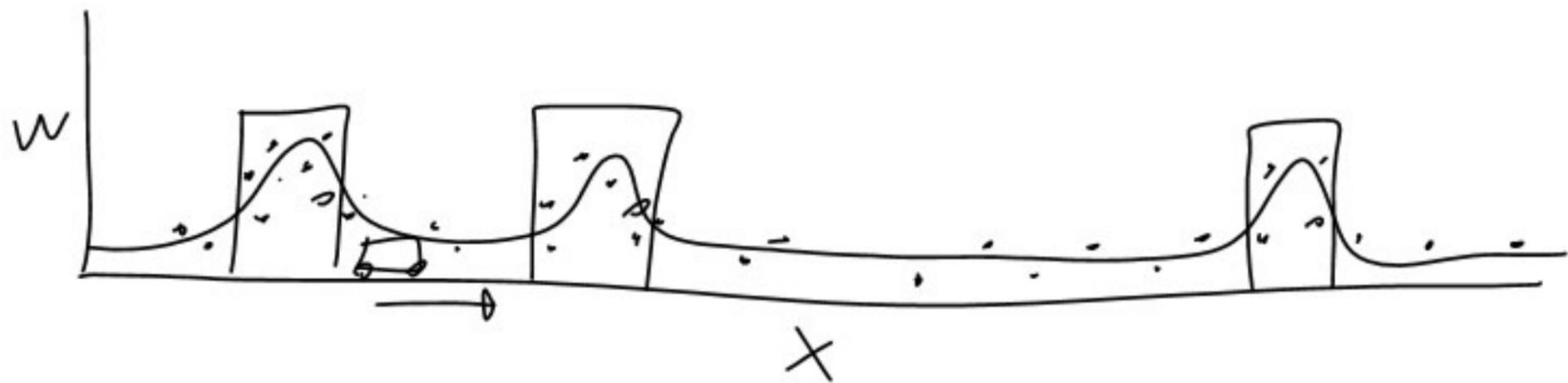
nonlinear

limited

scalable

scalable

scalable



Particle

Predict: move particles and add noise

$$x_t \sim p(x_t | x_{t-1}, u_t)$$

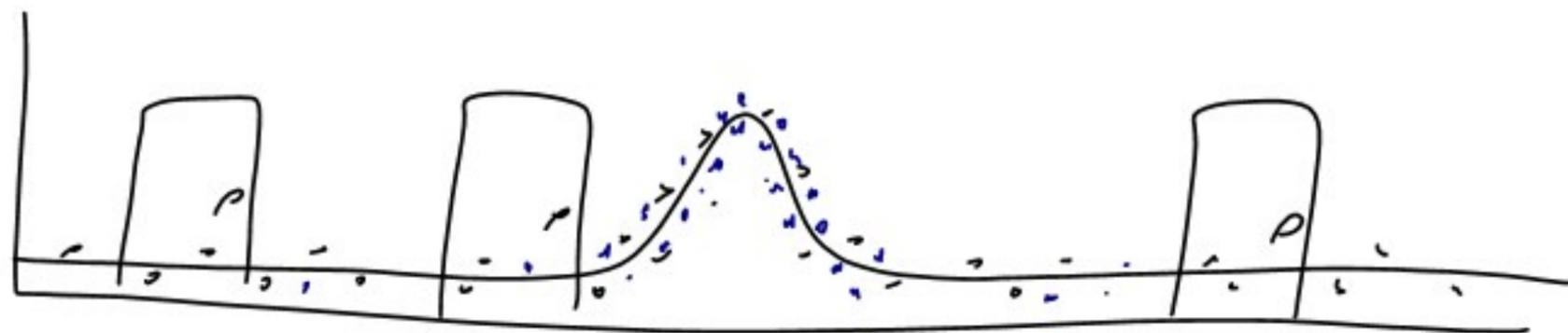
Sampling from motion model

Update:

importance factors or weights

$$w = p(x_t | z_t)$$

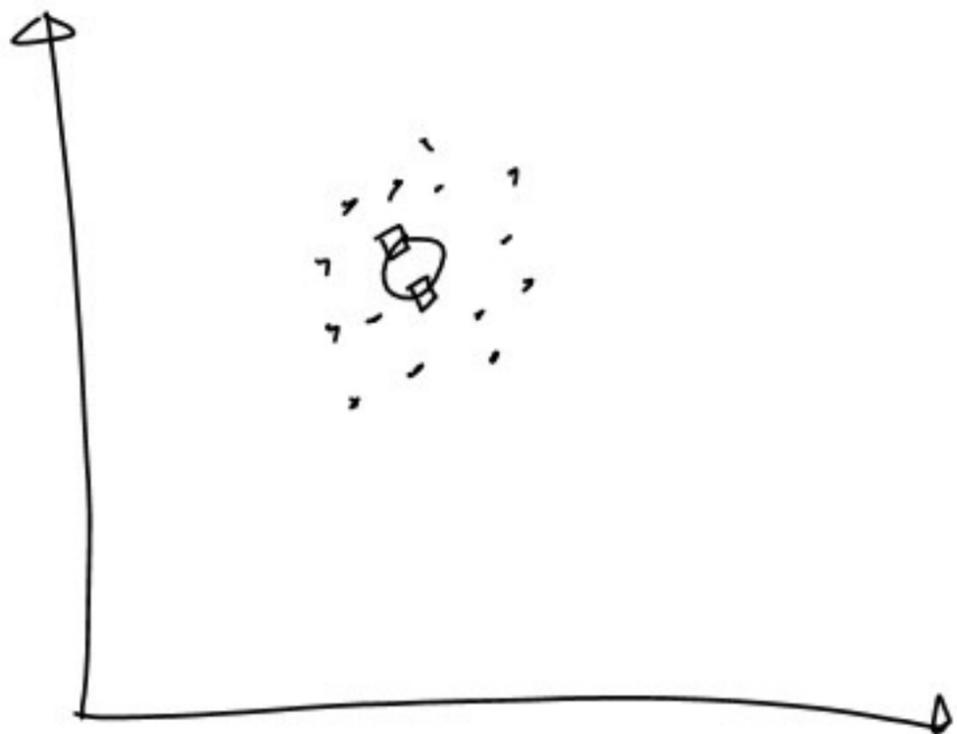
# Resampling



## Mean

normalize weights

$$E[X] = \sum_i w_i x_i$$



```
1: Algorithm Particle_filter( $\mathcal{X}_{t-1}, u_t, z_t$ ):
2:    $\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$ 
3:   for  $m = 1$  to  $M$  do
4:     sample  $x_t^{[m]} \sim p(x_t | u_t, x_{t-1}^{[m]})$  predict
5:      $w_t^{[m]} = p(z_t | x_t^{[m]})$  update
6:      $\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$ 
7:   endfor
8:   for  $m = 1$  to  $M$  do
9:     draw  $i$  with probability  $\propto w_t^{[i]}$ 
10:    add  $x_t^{[i]}$  to  $\mathcal{X}_t$ 
11:   endfor
12:   return  $\mathcal{X}_t$ 
```

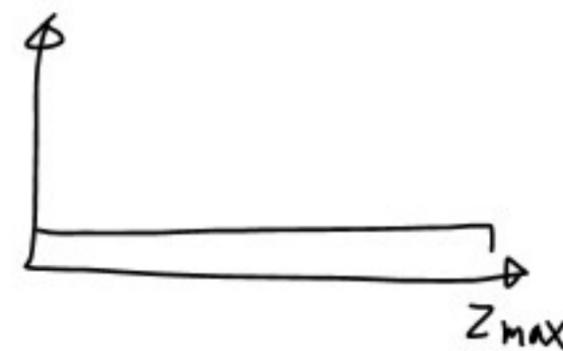
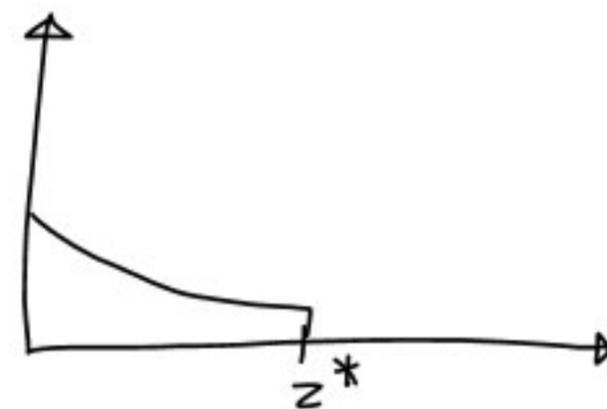
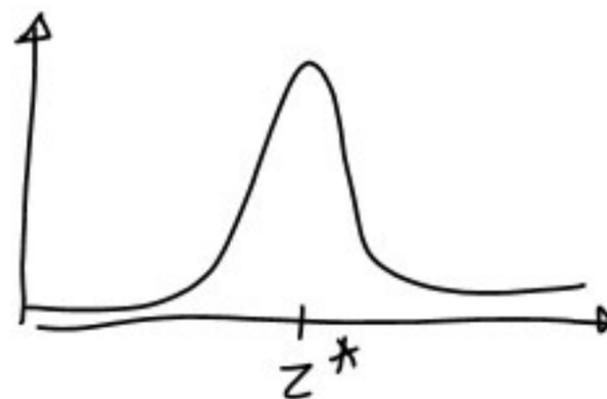
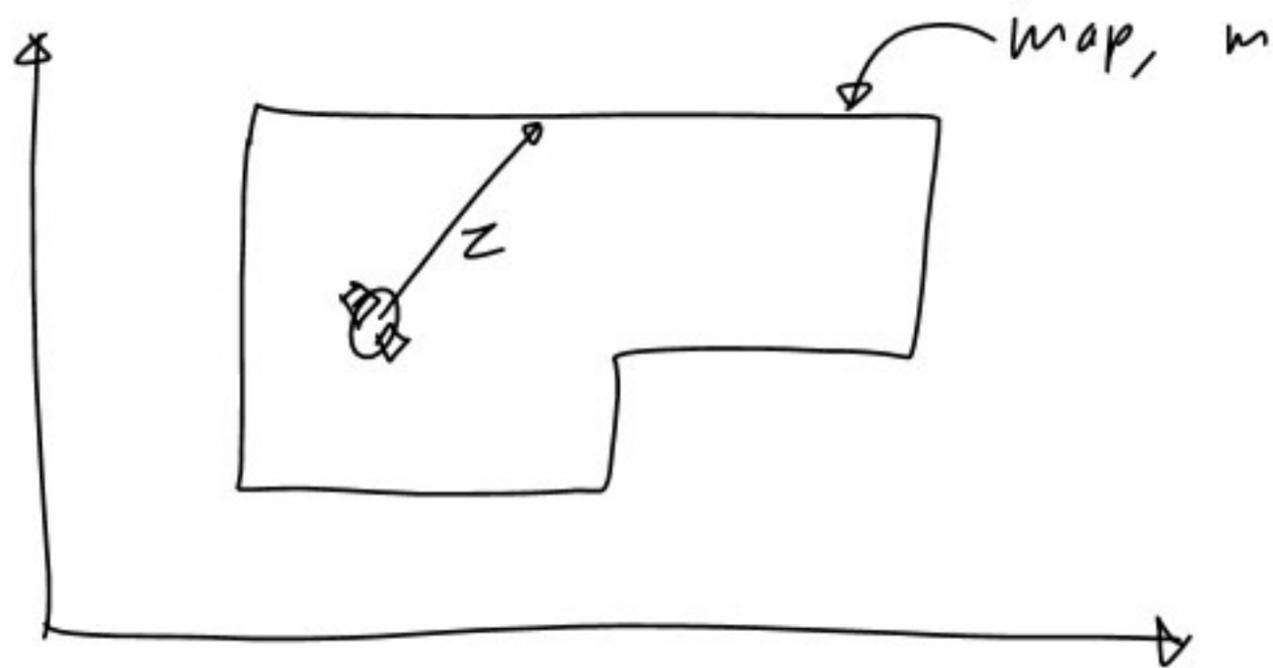
*resample*

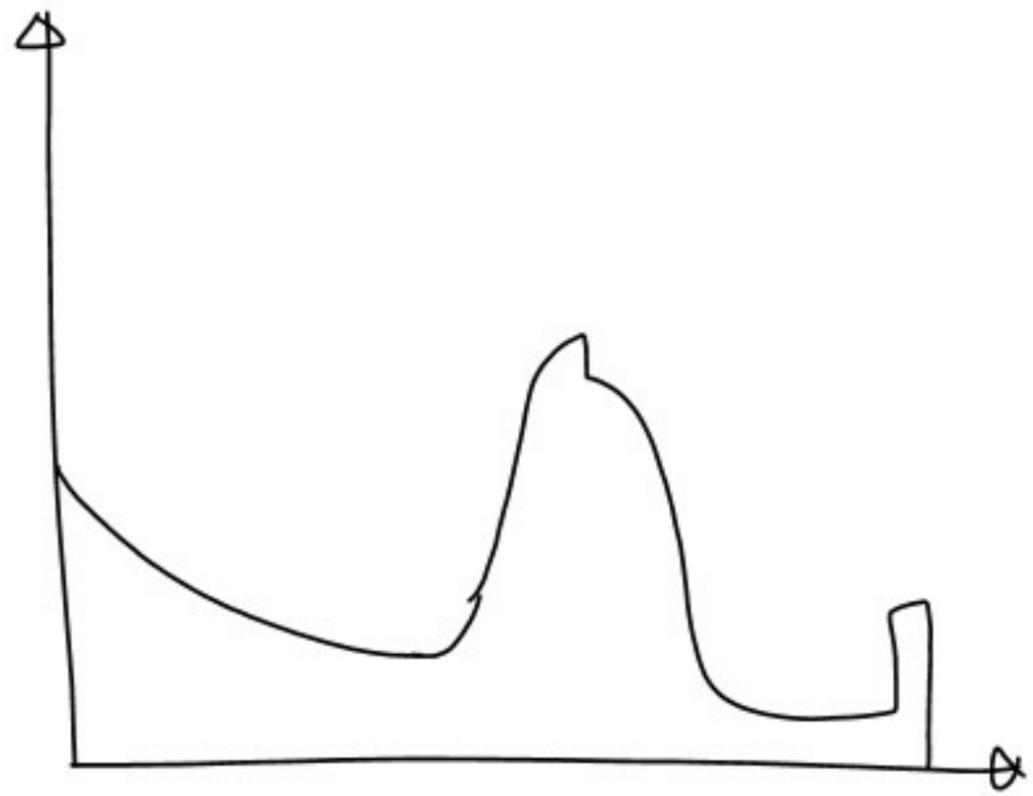
# Measurement Models

$$p(z|x, m)$$

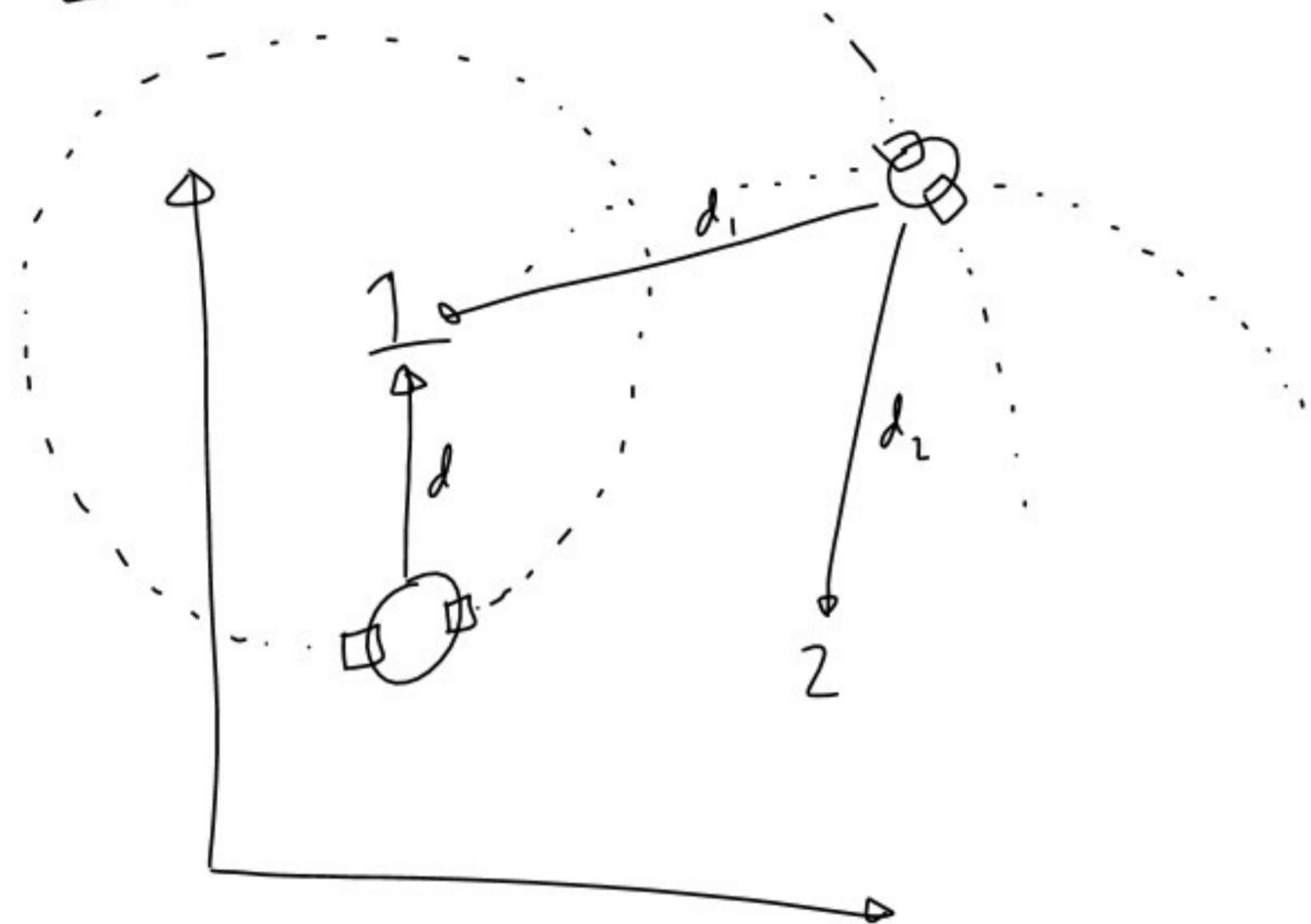
$z^*$  true measurement

## Ray Sensor





# Lab 3: Feature Based Localization



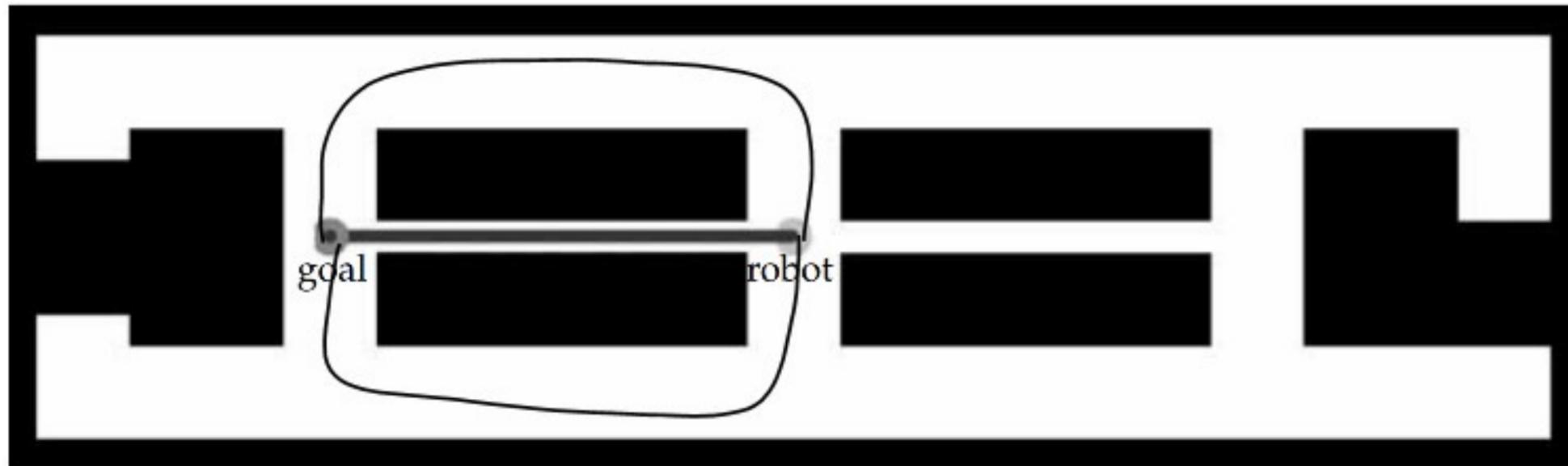
Fiducials

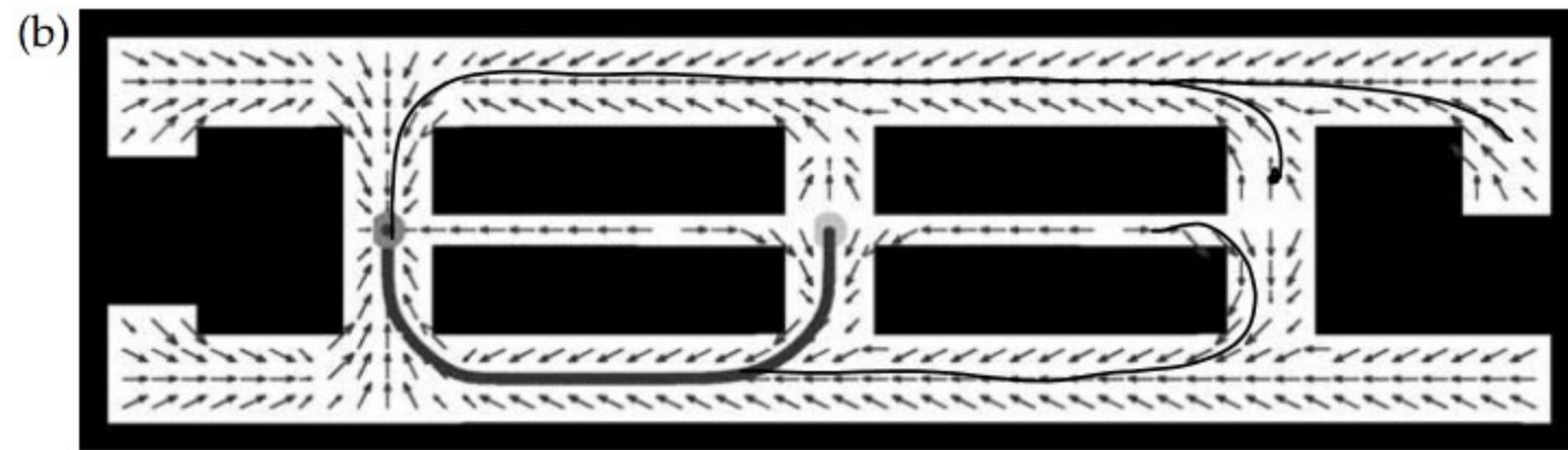
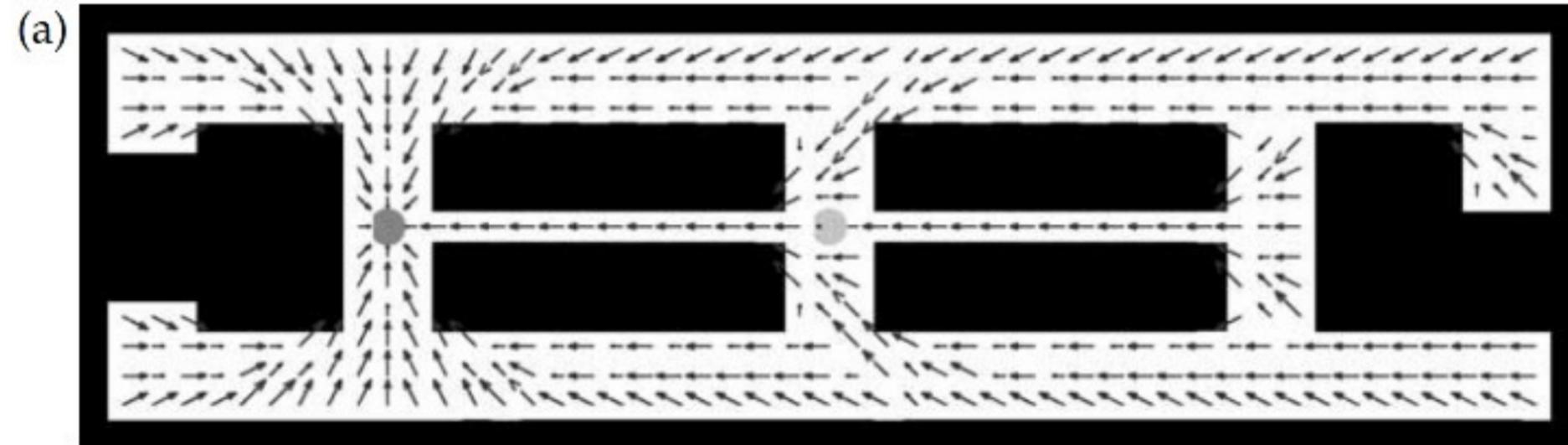
April Tags

Known Correspondence

# Path Planning

## Markov Decision Process





## Payoff Function

$$r(x, u) = \begin{cases} 100 & \text{if } x \text{ is goal} \\ -1 & \text{otherwise} \end{cases}$$

$$R_T = \sum_{\tau=1}^T \gamma^{\tau} r_{t+\tau}$$

$\gamma$  discount factor

Greedy  $T=1$

Finite horizon  $1 < T < \infty$

Infinite horizon  $T = \infty$   $0 < \gamma < 1$

# Value Function

$$V_1(x) = \gamma \max_u r(x, u)$$

$$V_2(x) = \gamma \max_u r(x, u) + \int V_1(x') p(x' | u, x) dx'$$

$$V_T(x) = \gamma \max_u r(x, u) + \int V_{T-1}(x') p(x' | u, x) dx'$$

1:     **Algorithm MDP\_discrete\_value\_iteration( ):**  
2:     *for*  $i = 1$  *to*  $N$  *do*  
3:          $\hat{V}(x_i) = r_{\min}$   
4:     *endfor*  
5:     *repeat until convergence*  
6:         *for*  $i = 1$  *to*  $N$  *do*  
7:             
$$\hat{V}(x_i) = \gamma \max_u \left[ r(x_i, u) + \sum_{j=1}^N \hat{V}(x_j) p(x_j | u, x_i) \right]$$
  
8:         *endfor*  
9:     *endrepeat*  
10:     *return*  $\hat{V}$

