Mapping and SLAM

Localization vs Mapping

Have map

Find location

Have location ← always uncertain

Find map

Simultaneous Localization and Mapping (SLAM)

Particle filter

Robot state $\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

map $\left(\begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}, \ldots \right)$

Example

10 fiducials

state vector size

$n = 3 + 2 \cdot 10 = 23$

5 particles along each axis

$5^{23} \simeq 10^{16}$ particles

use 32 bit float (4 bytes)

$23 \cdot 4 \cdot 10^{16} = 9.2 \times 10^{17}$ bytes

$= 8.57 \times 10^{8}$ TB

$= 817$ EB

Fast SLAM

Rao Blackwellized Particle Filter

$$\bar{X}_i = \left\{ \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}, \mu_1, P_1, \mu_2, P_2, \ldots \right\}$$

$\underbrace{\mu_1, P_1, \mu_2, P_2}_{\text{map}}$

EKF on each particle

$$\mu_i = \underset{I}{F}\mu_i + \underset{0}{B}u \Rightarrow \mu_i = \mu_i$$

$$P_i = \underset{I}{F} P_i \underset{I}{F}^T + \underset{0}{Q} = P_i$$

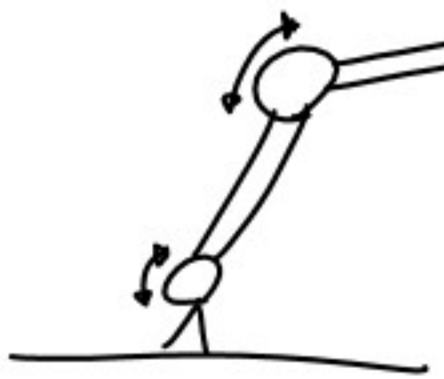
no prediction step

Standard Update Step

$$w = \sqrt{|2\pi Q'|}\, \exp\left(-\tfrac{1}{2}(z-\hat{z})\, Q'^{-1} (z-\hat{z})^T\right)$$

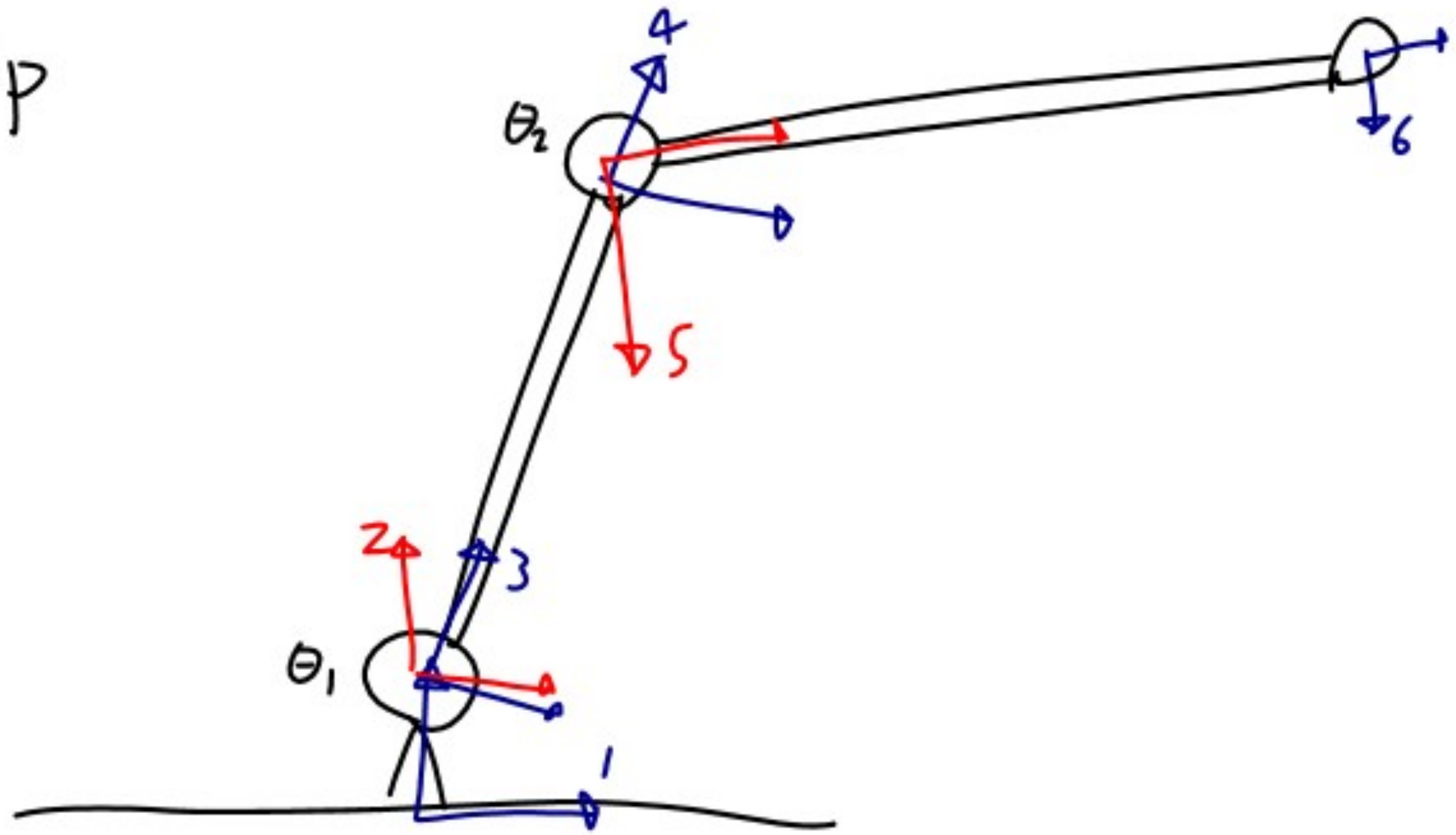
$$Q' = H^T P H + Q$$

Serial Kinematic Robots



$$^{1}P = {}^{2}P + P_{2,1}$$

$$^{2}P = {}^{2}_{3}R\,{}^{3}P$$



$$R = \begin{bmatrix} | & | & | \\ a & b & c \\ | & | & | \end{bmatrix}$$

$$|a| = |b| = |c| = 1$$

$$a \times b = c$$
$$b \times c = a$$
$$c \times a = b$$

$$Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$

$$C\theta = \cos(\theta)$$

$$S\theta = \sin(\theta)$$

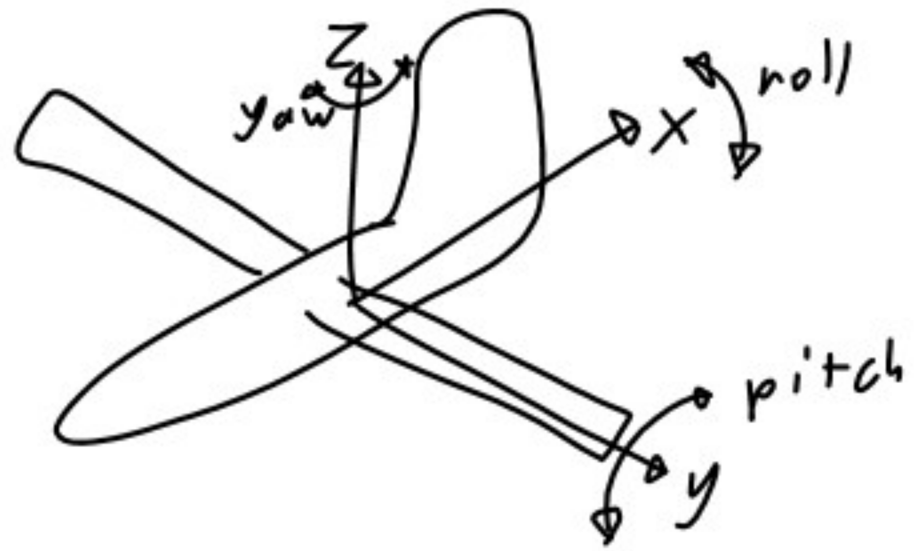
$$Rot(y, \theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

$$Rot(z, \theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Euler Angles

Roll Pitch Yaw

$$R_{rpy} = Rot(z, yaw)\, Rot(y, pitch)\, Rot(x, roll)$$



Rotation Inverses

$^{2}_{3}R$

$^{3}_{2}R = {}^{2}_{3}R^{-1} = {}^{2}_{3}R^{T}$

Homogeneous Transformations

$$ {}^{4}P = \begin{bmatrix} {}^{4}x \\ {}^{4}y \\ {}^{4}z \end{bmatrix} = \begin{bmatrix} {}^{4}x \\ {}^{4}y \\ {}^{4}z \\ 1 \end{bmatrix} $$

$$ {}^{2}_{4}T = \begin{bmatrix} \begin{bmatrix} & {}^{2}_{4}R & \end{bmatrix} & \begin{bmatrix} P_{4,2} \end{bmatrix} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} $$

$$ {}^{4}_{2}T = {}^{2}_{4}T^{-1} = \begin{bmatrix} \begin{bmatrix} & {}^{2}_{4}R^{T} & \end{bmatrix} & \begin{bmatrix} -{}^{2}_{4}R^{T} P_{4,2} \end{bmatrix} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix} $$