00.5 Binary and hexadecimal arithmetic

In order to perform arithmetic operations on binary and hexadecimal numbers, a straightforward method is to convert the numbers to decimal, operate arithmetically in the usual way, then convert the result back to binary or hex.
However, arithmetic with all numbers represented by positional numeral systems can be performed in a familiar manner. We demonstrate this, by example, with binary, but this method also applies for hexadecimal arithmetic.

Example 00.5 −1 — re: binary summation

Sum 1110_2 and 1100_2.

$$\begin{array}{r} {\scriptstyle 1\,1\ \ } \\ 01110 \\ +\,01100 \\ \hline 11010 \end{array}$$

1+1 = 10
1+1+1 = 10 + 1 = 11

Example 00.5 −2 — re: binary subtraction

Subtract 1010_2 from 1100_2.

$$\begin{array}{r} {\scriptstyle 0\ \ \ } \\ 1\cancel{1}00 \\ -\,1010 \\ \hline 0010 \end{array}$$

$10 - 1_2$
$(2-1)_{10} = 1_{10} = 1_2$

Example 00.5 −3 — re: binary multiplication

Multiply 1100_2 and 1010_2.

$$\begin{array}{r} 1100 \\ \times\,1010 \\ \hline 0000 \\ 1100\ \ \\ 0000\ \ \ \ \\ +\ 1100\ \ \ \ \ \ \\ \hline 1111000_2 \end{array}$$

$1111000_2 = 8+16+32+64 = 120_{10}$

$$\begin{array}{r} 12 \\ \times 10 \\ \hline 120 \end{array}$$

$$\begin{array}{r} 0101 \\ \times\,0100 \\ \hline 0000 \\ 0000\ \ \\ 0101\ \ \ \ \\ +\ 0000\ \ \ \ \ \ \\ \hline 0010100 \end{array}$$

$0010100 = 4 + 16 = 20_{10}$

$$\begin{array}{r} 5 \\ \times 4 \\ \hline 20 \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\,1\ \ \ } \\ 0110 \\ +\,1011 \\ \hline \cancel{1}0001 \end{array}$$

$0001_2 = 1_{10}$
$10001_2 = 17_{10}$

$$\begin{array}{r} 0 \\ \times\,011 \\ -\,0110 \\ \hline 0101 \end{array}$$

$$\begin{array}{r} 11 \\ -6 \\ \hline 5 \end{array}$$

$0110_2 = 2+4 = 6_{10}$
$1011_2 \to 0100 \to 0101_2 = 1+4 = 5$
$= -5_{10}$

$$\begin{array}{r} 6 \\ +\ -5 \\ \hline 1 \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\,1\ \ } \\ 0111 \\ +\ 0111 \\ \hline 1110 \end{array}$$

$0111_2 = 1+2+4 = 7_{10}$

$$\begin{array}{r} 7 \\ +\ 7 \\ \hline 14 \end{array}$$

$1110_2 \to 0001_2 \to 0010_2$
$0010_2 = 2_{10}$
$1110_2 = -2_{10}$
$7+7 = -2$