06.4 The biquad cascade

Although we could implement Eq. 4 as shown, the sensitivity of the output to the coefficients leads to numerical inaccuracies as the order of the system N becomes large. We will solve this problem by breaking the Nth order system it into a series of n_s second-order systems. The technique is called a biquad cascade and is illustrated in Figure 06.1. Notice that the output of each second-order section (biquad)[4] is the input to the subsequent section. Each biquad implements the same second-order difference equation, but with different coefficients, inputs, and outputs. For example, the current output $y_i(n)$ from the ith section would be:

4. "Biquad" is short for "biquadratic." The biquad transfer function has second-order polynomials in both numerator and denominator.

$H(s) = G(s)\,C(s)\,T(s)$

$$y_i(n) = \frac{1}{a_{0_i}}\Big(b_{0_i}x_i(n) + b_{1_i}x_i(n-1) + b_{2_i}x_i(n-2) + \\ - a_{1_i}y_i(n-1) - a_{2_i}y_i(n-2)\Big). \tag{1}$$

Of course, a first or second order transfer function would require only one biquad. Depending on the value of N, some of the coefficients of at least one biquad may be zero. We will implement a function to handle any value of N. There are a variety of algorithms for breaking a transfer function into biquadric sections. Matlab's Signal Processing Toolbox contains a

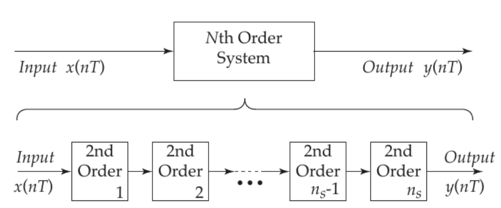

Figure 06.1: a biquad cascade.

function `tf2sos` (transfer function to second order sections) for this purpose.

$$\begin{bmatrix} b_{01} & b_{11} & b_{21} & 1 & a_{11} & a_{21} \\ b_{02} & b_{12} & b_{22} & 1 & a_{12} & a_{22} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{0n_s} & b_{1n_s} & b_{2n_s} & 1 & a_{1n_s} & a_{2n_s} \end{bmatrix}$$

$$H(z) = \prod_{k=1}^{n_s} \frac{b_{0k} + b_{1k}z^{-1} + b_{2k}z^{-2}}{1 + a_{1k}z^{-1} + a_{2k}z^{-2}}$$