Heat Transfer

$$\dot{Q} = -K \nabla T$$

$$\dot{Q} = -KA \frac{\partial T}{\partial x}$$

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

$$\dot{Q} = \frac{-KA(T_2 - T_1)}{L}$$

$$R = \frac{L}{KA}$$

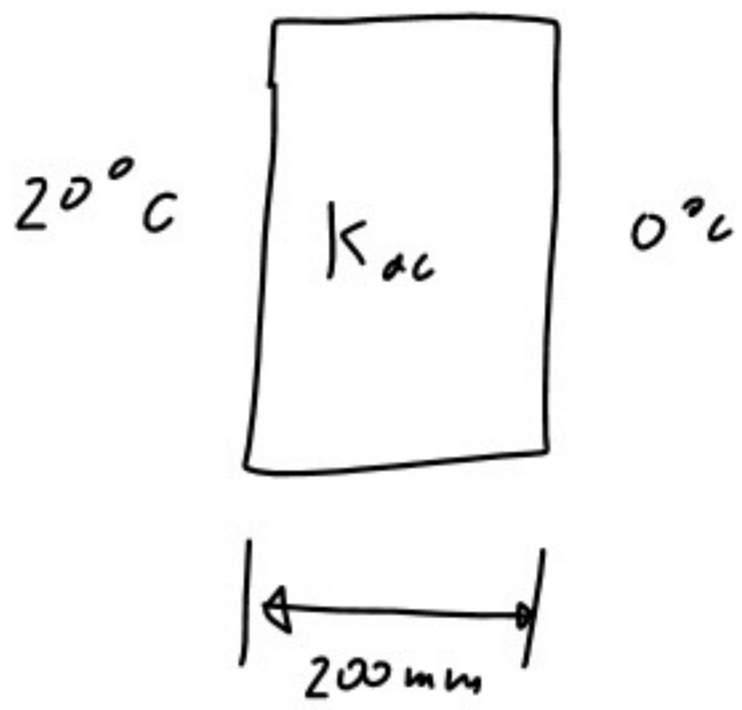
$$= \frac{1}{hA}$$

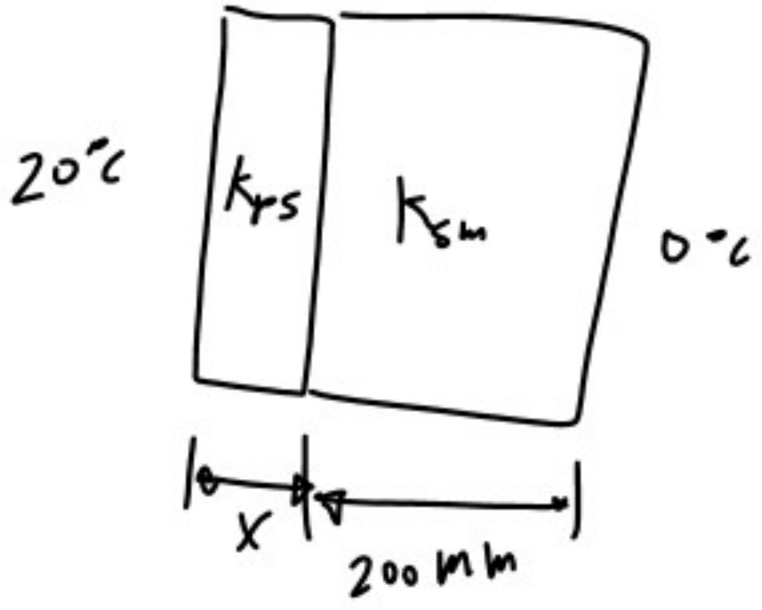
$$\dot{Q} = \frac{\Delta T}{\Sigma R}$$

Cylindrical wall

$$\dot{Q} = \frac{2\pi KL(T_1 - T_2)}{\ln(r_2/r_1)}$$

3.2 A new building to be located in a cold climate is being designed with a basement that has an $L = 200$-mm-thick wall. Inner and outer basement wall temperatures are $T_i = 20°C$ and $T_o = 0°C$, respectively. The architect can specify the wall material to be either aerated concrete block with $k_{ac} = 0.15$ W/m · K, or stone mix concrete. To reduce the conduction heat flux through the stone mix wall to a level equivalent to that of the aerated concrete wall, what thickness of extruded polystyrene sheet must be applied onto the inner surface of the stone mix concrete wall? Floor dimensions of the basement are 20 m × 30 m, and the expected rental rate is $50/m²/month. What is the yearly cost, in terms of lost rental income, if the stone mix concrete wall with polystyrene insulation is specified?



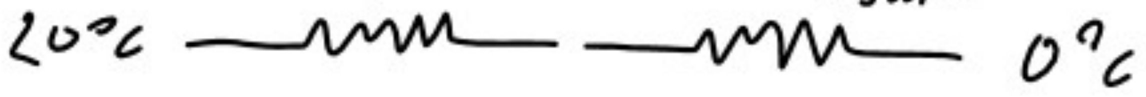


$K_{sm} = 1.4 \ \frac{W}{mK}$

$K_{ps} = 0.027 \ \frac{W}{mK}$

$R_{ac} = \frac{L}{K_{ac} A}$

$R_{ps} = \frac{x}{K_{ps} A}$ $\quad R_{sm} = \frac{L}{K_{sm} A}$



$$R_{ac} = R_{ps} + R_{sm}$$

$$\frac{L}{K_{ac} A} = \frac{x}{K_{ps} A} + \frac{L}{K_{sm} A}$$

$$\frac{L}{K_{ac}} = \frac{x}{K_{ps}} + \frac{L}{K_{sm}}$$

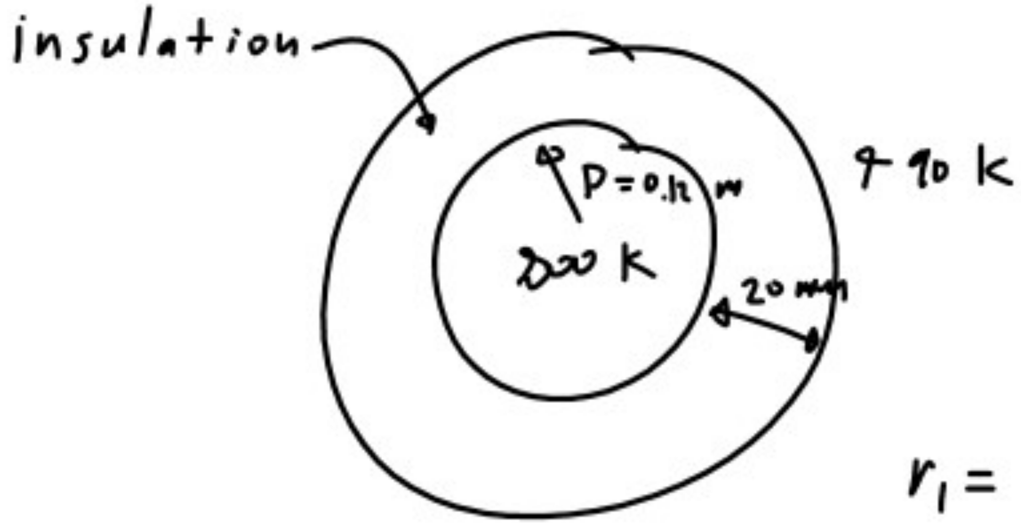
$$\frac{0.2m}{0.15 \frac{\cancel{W}}{\cancel{m}K}} = \frac{x}{0.027 \frac{\cancel{W}}{\cancel{m}K}} + \frac{0.2m}{1.4 \frac{\cancel{W}}{\cancel{m}K}}$$

$$1.33m = 37.04x + 0.14m$$

$$\frac{1.33m - 0.14m}{37.04} = 0.032m = \boxed{32mm = x}$$

3.39 A steam pipe of 0.12-m outside diameter is insulated with a layer of calcium silicate.

(a) If the insulation is 20 mm thick and its inner and outer surfaces are maintained at $T_{s,1} = 800$ K and $T_{s,2} = 490$ K, respectively, what is the rate of heat loss per unit length (q') of the pipe?



$k = 0.089 \frac{W}{mK}$

$$r_1 = \frac{0.12\,m}{2} = \frac{120\,mm}{2} = 60\,mm$$

$$r_2 = 60 + 20\,mm = 80\,mm$$

$$\dot{Q} = \frac{2\pi k L (T_1 - T_2)}{\ln(r_2/r_1)} = \frac{2\pi\, 0.089 \frac{W}{m\cancel{K}}\, L\, (800\cancel{K} - 490\cancel{K})}{\ln\left(\frac{80\,\cancel{mm}}{60\,\cancel{mm}}\right)}$$

$$= 603\, L\, \frac{W}{m}$$

$$\frac{\dot{Q}}{L} = 603 \frac{W}{m}$$

Fins

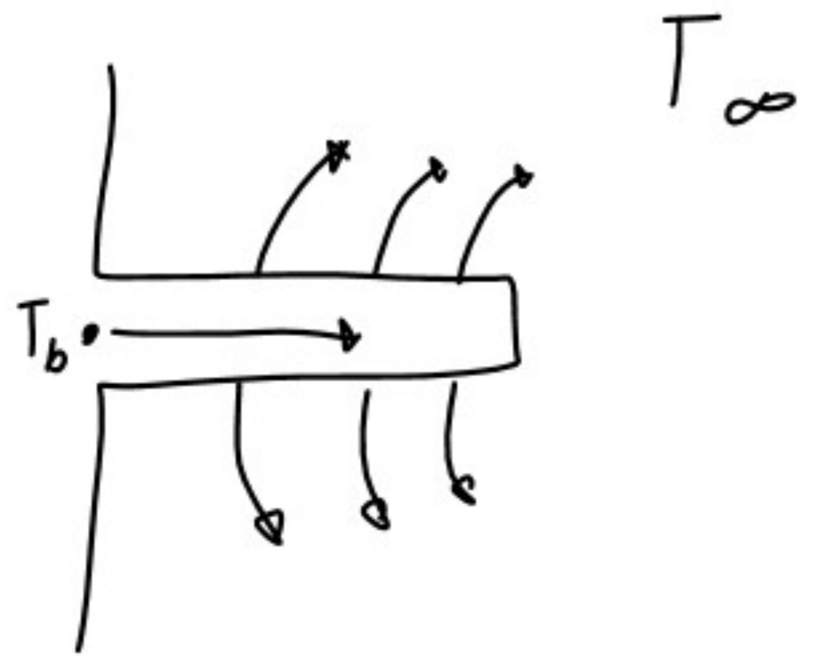
$$\dot{Q} = \sqrt{hPKA_c}\,(T_b - T_\infty)\tanh(mL_c)$$

$$L_c = L + \frac{A_c}{P}$$

$$m = \sqrt{\frac{hP}{KA_c}}$$

P perimeter

A_c cross-sectional area

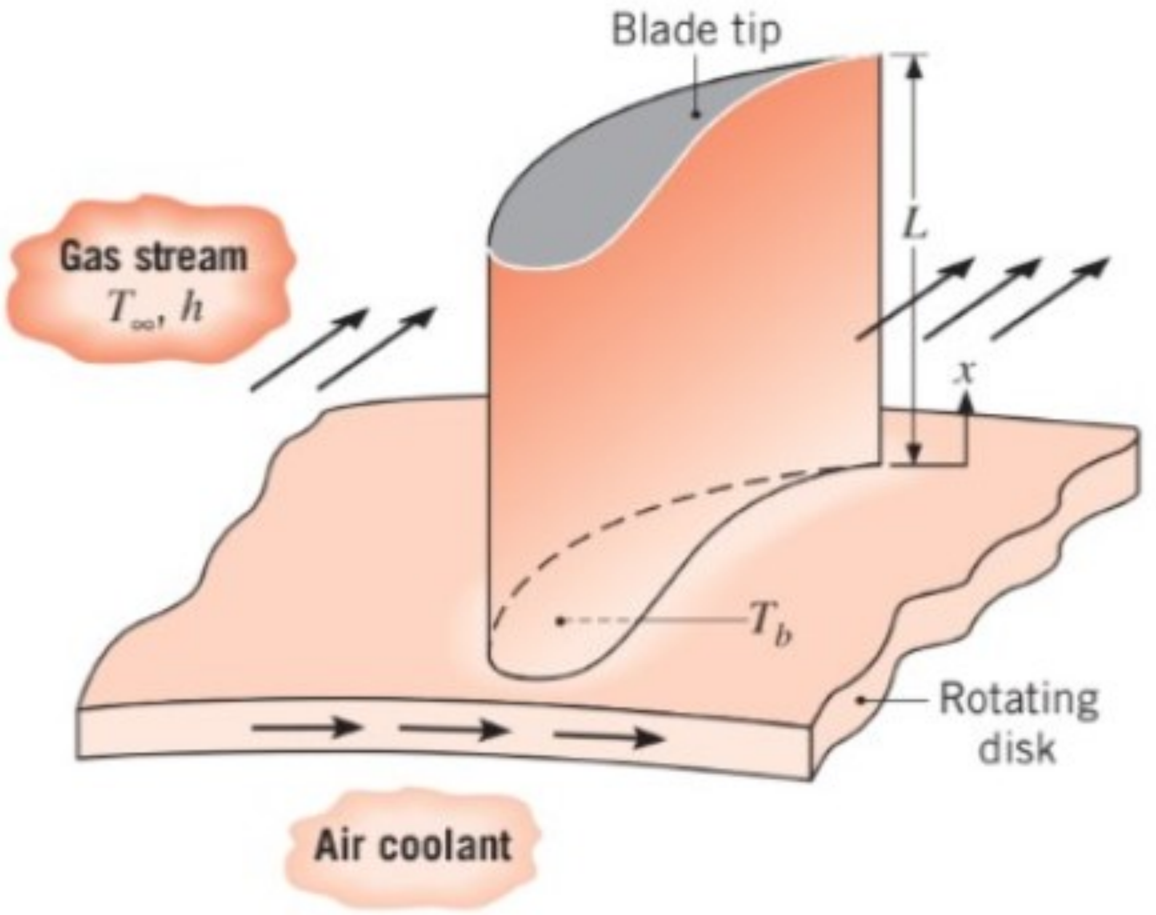


3.99 Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at $T_\infty = 1200°C$ and maintains a convection coefficient of $h = 250\ W/m^2 \cdot K$ over the blade.

The blades, which are fabricated from Inconel, $k \approx 20\ W/m \cdot K$, have a length of $L = 50$ mm. The blade profile has a uniform cross-sectional area of $A_c = 6 \times 10^{-4}\ m^2$ and a perimeter of $P = 110$ mm. A proposed blade-cooling scheme, which involves routing air through the supporting disc, is able to maintain the base of each blade at a temperature of $T_b = 300°C$.

(a) ~~If the maximum allowable blade temperature is 1050°C and the blade tip may be assumed to be adiabatic, is the proposed cooling scheme satisfactory?~~

(b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?



$$\dot{Q} = \sqrt{hPKA_c}\,(T_b - T_\infty)\tanh(mL_c)$$

$$L_c = L + \frac{A_c}{P} = 0.05\,m + \frac{6\times10^{-4}\,m^2}{0.11\,\cancel{m}} = 0.055\,m$$

$$= \sqrt{250\,\frac{W}{\cancel{m^2}K}\;0.11\,\cancel{m}\;20\,\frac{W}{\cancel{m}K}\;6\times10^{-4}\,\cancel{m^2}}\;(300\,^\circ C - 1200\,^\circ C)\;\tanh\!\left(47.9\,\frac{1}{m}\;0.055\,m\right)$$

$$= \sqrt{0.33\,\frac{W^2}{K^2}}\;(-900\,K)\;0.99$$

$$= 0.57\,\frac{W}{\cancel{K}}\;(-900\,\cancel{K})\;0.99 = -512\,W$$

$$m = \sqrt{\frac{hP}{KA_c}}$$

$$= \sqrt{\frac{250\,\frac{\cancel{W}}{m^2\cancel{K}}\;0.11\,\cancel{m}}{20\,\frac{\cancel{W}}{\cancel{m}\cancel{K}}\;6\times10^{-4}\,m^2}}$$

$$= \sqrt{2292\,{}^{1}/_{m^2}} = 47.9\,{}^{1}/_{m}$$

Transient conduction

Lumped Capacitance

$$Bi = \frac{hV}{KA_s} = \frac{hL_c}{K}$$

Biot Number

$$L_c = \frac{V}{A_s}$$

$$Bi << 1$$

$$T - T_\infty = (T_i - T_\infty) e^{-\beta t}$$

$$\beta = \frac{hA_s}{\rho V c_p} = \frac{1}{\tau}$$

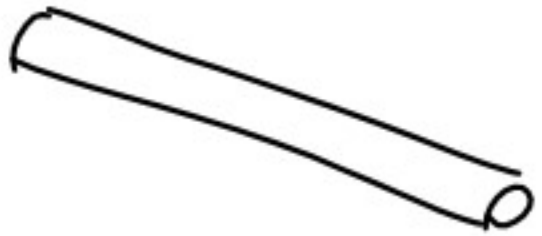
A_s surface area

V volume

ρ density

c_p heat capacity

5.15 Carbon steel (AISI 1010) shafts of 0.1-m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of 100 $W/m^2 \cdot K$. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?



$$k = 51.2 \frac{W}{m K}$$

$$\rho = 7832 \, kg/m^3$$

$$c_p = 541 \, J/kg\,K$$

$$Bi = \frac{hV}{k A_s} = \frac{h L_c}{k} = \frac{100 \frac{\cancel{W}}{\cancel{m^2 K}} 0.025 \cancel{m}}{51.2 \frac{\cancel{W}}{\cancel{m K}}} = 0.0488 << 1$$

$$L_c = \frac{V}{A_s} = \frac{\cancel{\pi} r^2 \cancel{L}}{\cancel{\pi} D \cancel{L}} = \frac{r^2}{D} = \frac{r^2}{2r} = \frac{r}{2} = \frac{0.05 m}{2} = 0.025 m$$

$$\beta = \frac{hA_s}{\rho V c_p} = \frac{h}{\rho L_c c_p} = \frac{100\ \frac{W}{\cancel{m^2 K}}}{7832\ \frac{\cancel{kg}}{\cancel{m^3}}\ 0.025\cancel{m}\ 541\ \frac{J}{\cancel{kg}\ K}} = 9.44\times10^{-4}\ \frac{\cancel{W}}{\cancel{J}}\ \frac{J}{\cancel{W}\cdot s}$$

$$T - T_\infty = (T_i - T_\infty)e^{-\beta t}$$

$$800\,K - 1200\,K = (300\,K - 1200\,K)\,e^{-9.44\times10^{-4}\frac{1}{s}t}$$

$$0.44 = e^{-9.44\times10^{-4}\frac{1}{s}t}$$

$$\ln(0.44) = -9.44\times10^{-4}\frac{1}{s}t$$

$$t = 859\,s = \boxed{14.3\ min}$$

7.8 Consider laminar, parallel flow past an isothermal flat plate of length L, providing an average heat transfer coefficient of $\bar{h}_L$. If the plate is divided into N smaller plates, each of length $L_N = L/N$, determine an expression for the ratio of the heat transfer coefficient averaged over the N plates to the heat transfer coefficient averaged over the single plate, $\bar{h}_{L,N}/\bar{h}_{L,1}$.

13.14 A drying oven consists of a long semicircular duct of diameter $D = 1.5$ m.

Materials to be dried cover the base of the oven while the wall is maintained at 1200 K. What is the drying rate per unit length of the oven (kg/s · m) if a water-coated layer of material is maintained at 350 K during the drying process? Blackbody behavior may be assumed for the water surface and the oven wall.