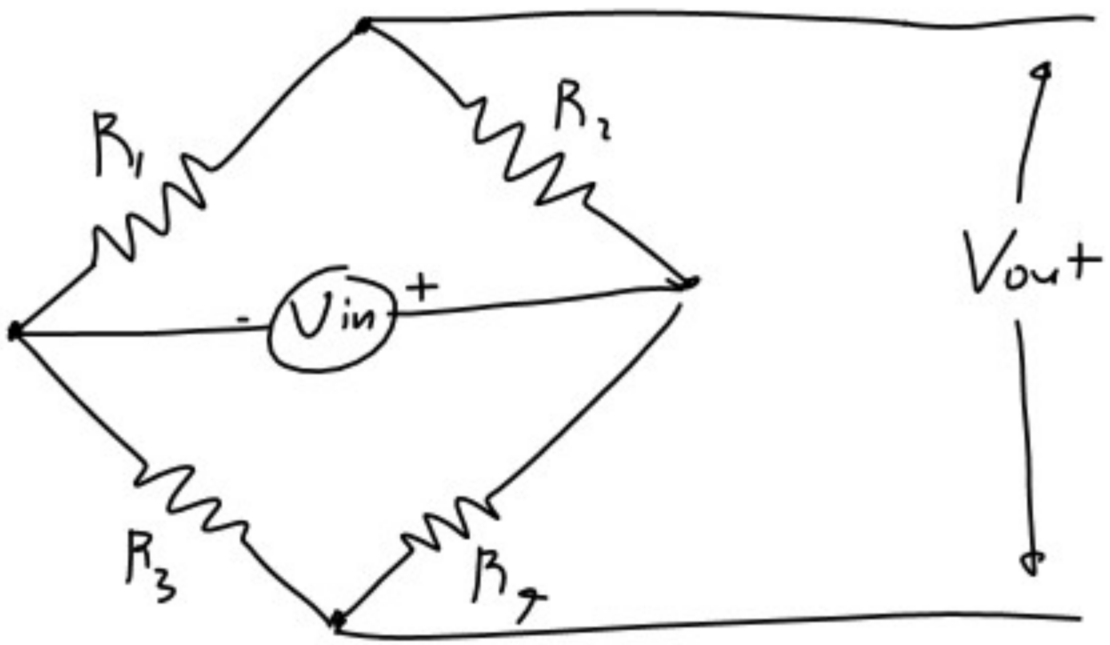
# Wheatstone Bridge



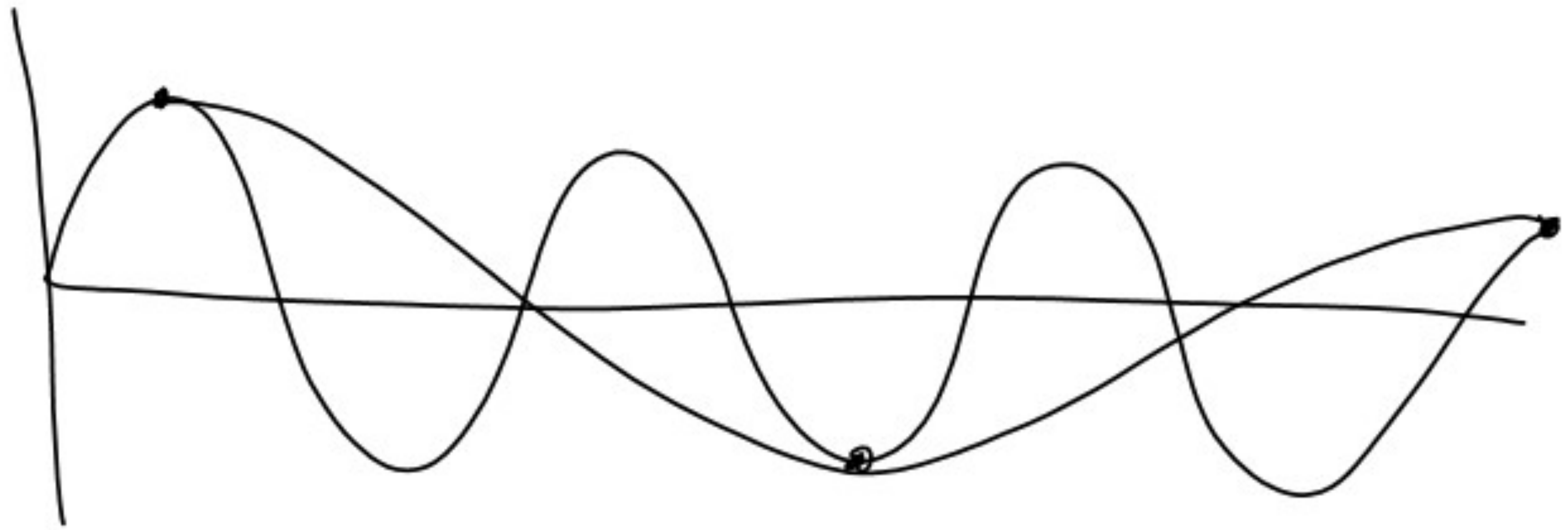

$$R_1 = R_2 = R_3 = R_4 \qquad V_{out} = 0$$

$$R_1 = R_2 = R_3 = R \qquad R_4 = R + \Delta R$$

$$V_{out} = \frac{\Delta R}{4R} V_{in}$$

# Sampling Theory

$$\frac{1}{\Delta t} = f_s > 2f_I$$



# Analog to Digital Conversion (ADC)

$$\varepsilon_v = \frac{V_H - V_L}{2^n} \qquad V = \varepsilon_v N + V_L$$

**Problem 6.9** Y5 As part of the control of a process, an analog signal is to be digitized at uniform intervals.

- The continuous signal is band-limited (narrow frequency spectrum), with no significant frequency components greater than 15 kHz.
- The dynamic range of the signal is ±5 V.
- ADCs are available with resolutions of 4, 6, 8, 10, $\boxed{12}$, ... bits.
- The accuracy of the ADC precision voltage reference (referred to the input) is ±0.0002 V.
- The maximum allowable total error is ±0.002 V.

Suggest appropriate values for:

a. The ADC resolution
b. The minimum sampling rate
c. ~~The maximum aperture (or conversion response) time~~

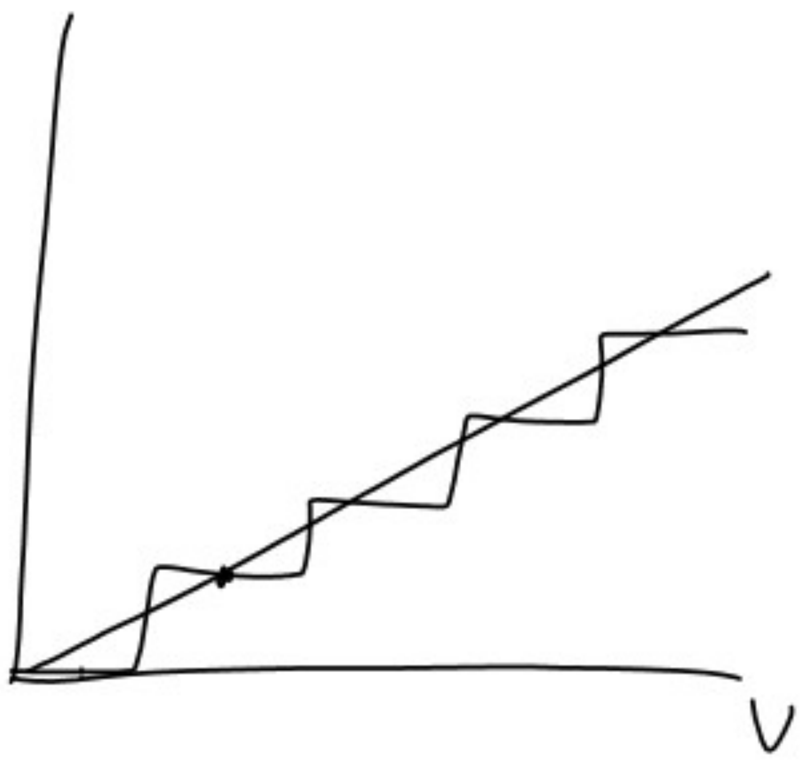
$$\varepsilon_v = \frac{V_H - V_L}{2^n}$$

$$0.004 = \frac{5-(-5)}{2^n}$$

$$2^n = \frac{10}{0.004} = 2500$$

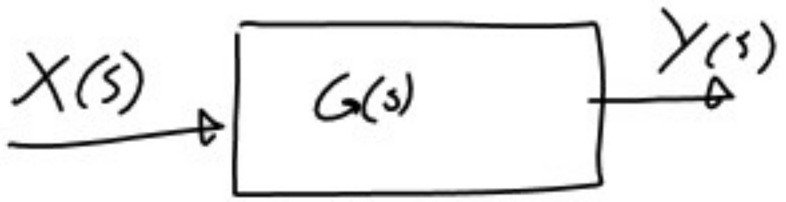
$$2^{12} = 4096$$

$$f_s = 2f_I = 2 \cdot 15\ \text{kHz} = 30\ \text{kHz}$$



# Transfer Functions

$$G(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n}{b_0 + b_1 s + b_2 s^2 + \cdots + b_m s^m}$$




$$\lim_{t \to 0} g(t) = \lim_{s \to \infty} sG(s)$$

$$\lim_{t \to \infty} g(t) = \lim_{s \to 0} sG(s)$$

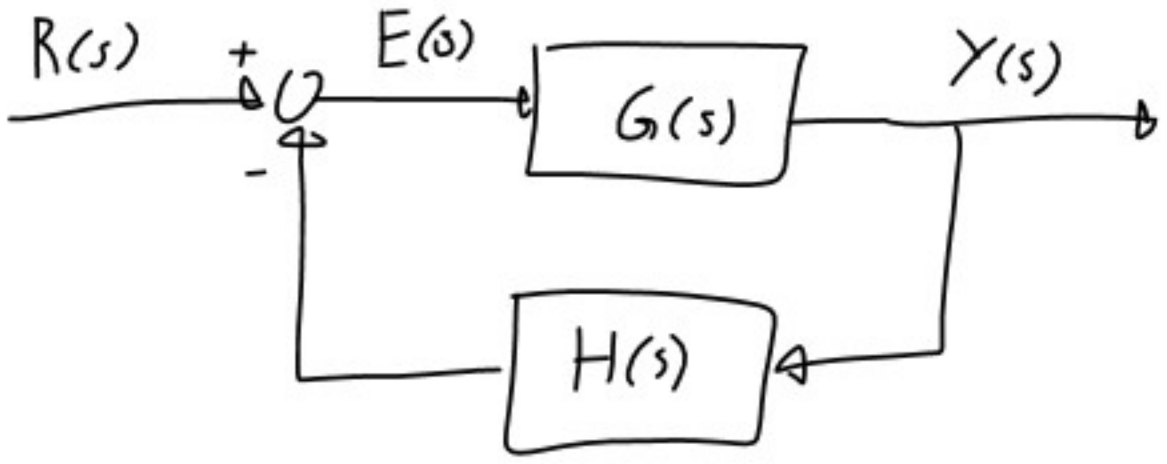
$$\mathcal{L}(y(t)) = Y(s)$$

$$y(t) = \mathcal{L}^{-1}(Y(s))$$

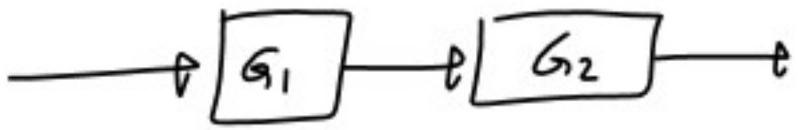
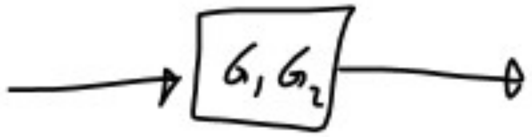
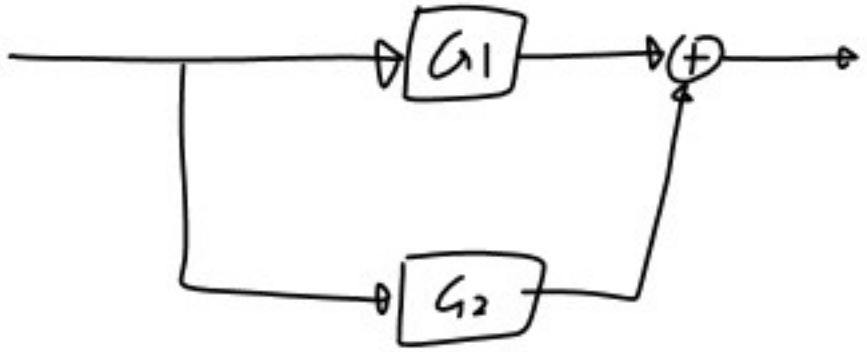
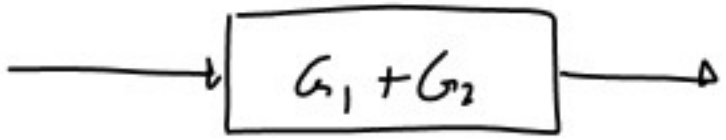
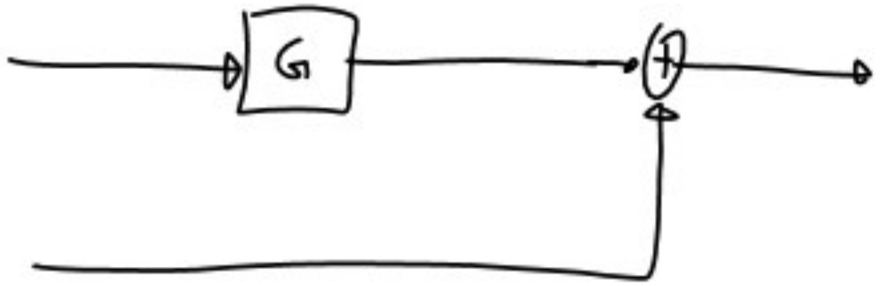
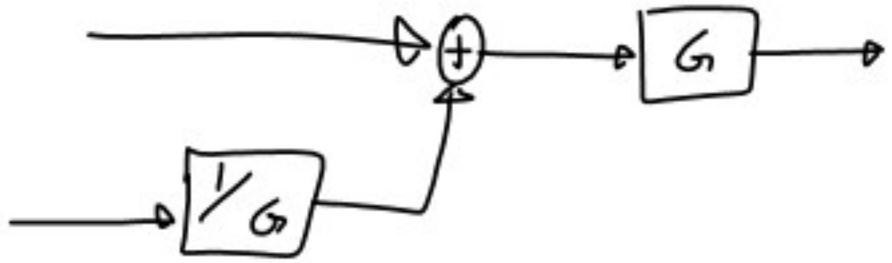
$$\mathcal{L}(y(t)) = \int_0^\infty y(t) e^{-st} \, dt \qquad s = \sigma + i\omega$$

$$\mathcal{F}(y(t)) = \int_{-\infty}^{\infty} y(t) e^{-i 2\pi \xi t} \, dt$$

# Feedback




$$G_{CL}(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$


G1
G2

G1G2

G1
+
G2

G1 + G2

G
+

+
G
1/G

**2.** Find the closed-loop transfer function, $T(s) = C(s)/R(s)$ for the system shown in Figure P5.2, using block diagram reduction. [Section: 5.2]

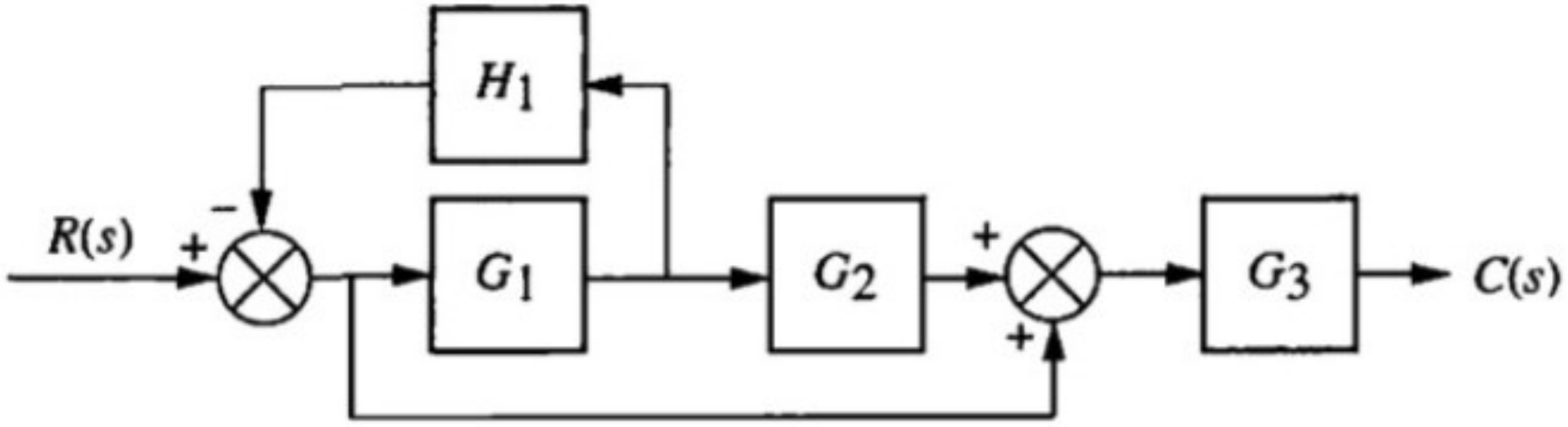
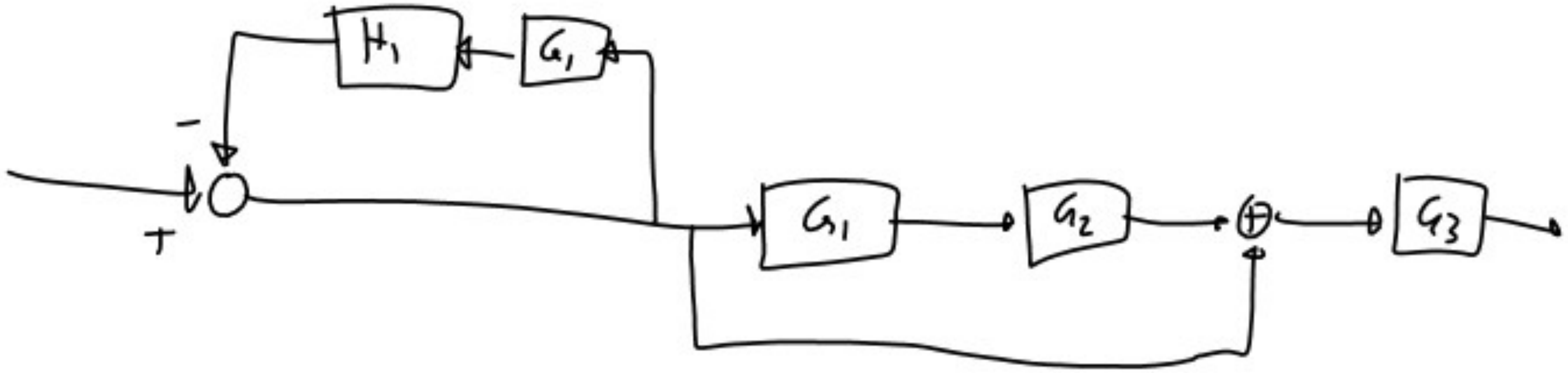
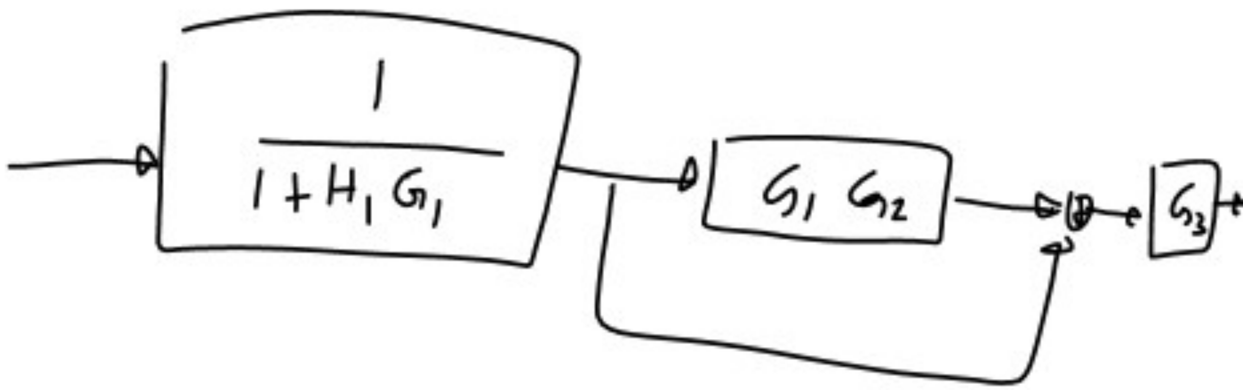



FIGURE P5.2



# Steady State Error

$$e_{ss}(t) = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} r(t) - y(t)$$

$$= \lim_{s\to 0} sE(s) = \lim_{s\to 0} s(R(s) - Y(s))$$

# Error Constants

| | input | $e_{ss}$ |
|---|---|---|
| $K_p = \lim_{s\to 0} G(s)$ | unit step | $\frac{1}{1+K_p}$ |
| $K_v = \lim_{s\to 0} sG(s)$ | unit ramp | $\frac{1}{K_v}$ |
| $K_a = \lim_{s\to 0} s^2 G(s)$ | parabolic | $\frac{1}{K_a}$ |

1. For the unity feedback system shown in Figure P7.1, where

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$$G(s) = \frac{450(s+8)(s+12)(s+15)}{s(s+38)(s^2+2s+28)}$$

find the steady-state errors for the following test inputs: $25u(t)$, $37tu(t)$, $47t^2u(t)$. [Section: 7.2]

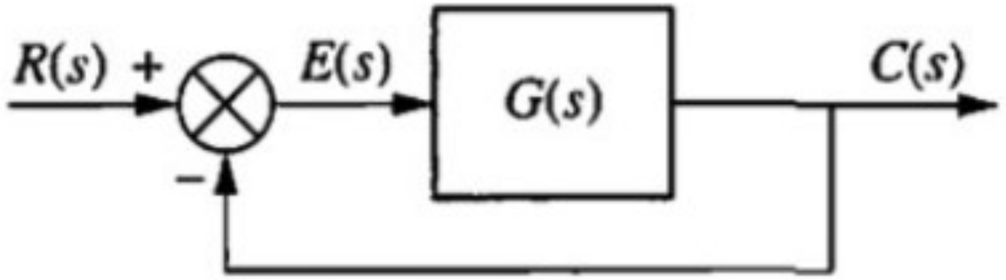
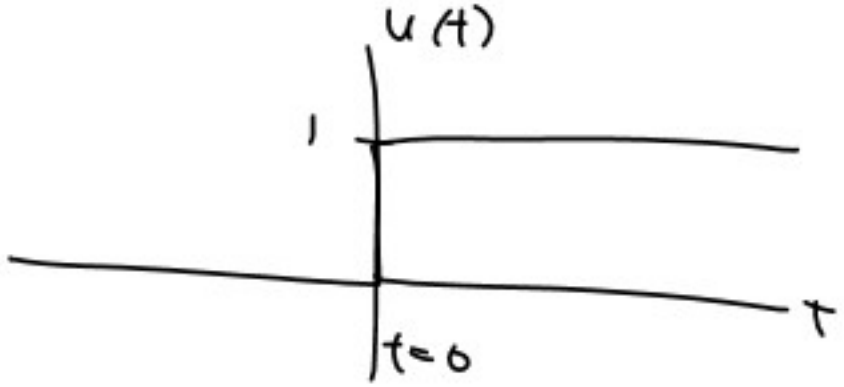



FIGURE P7.1



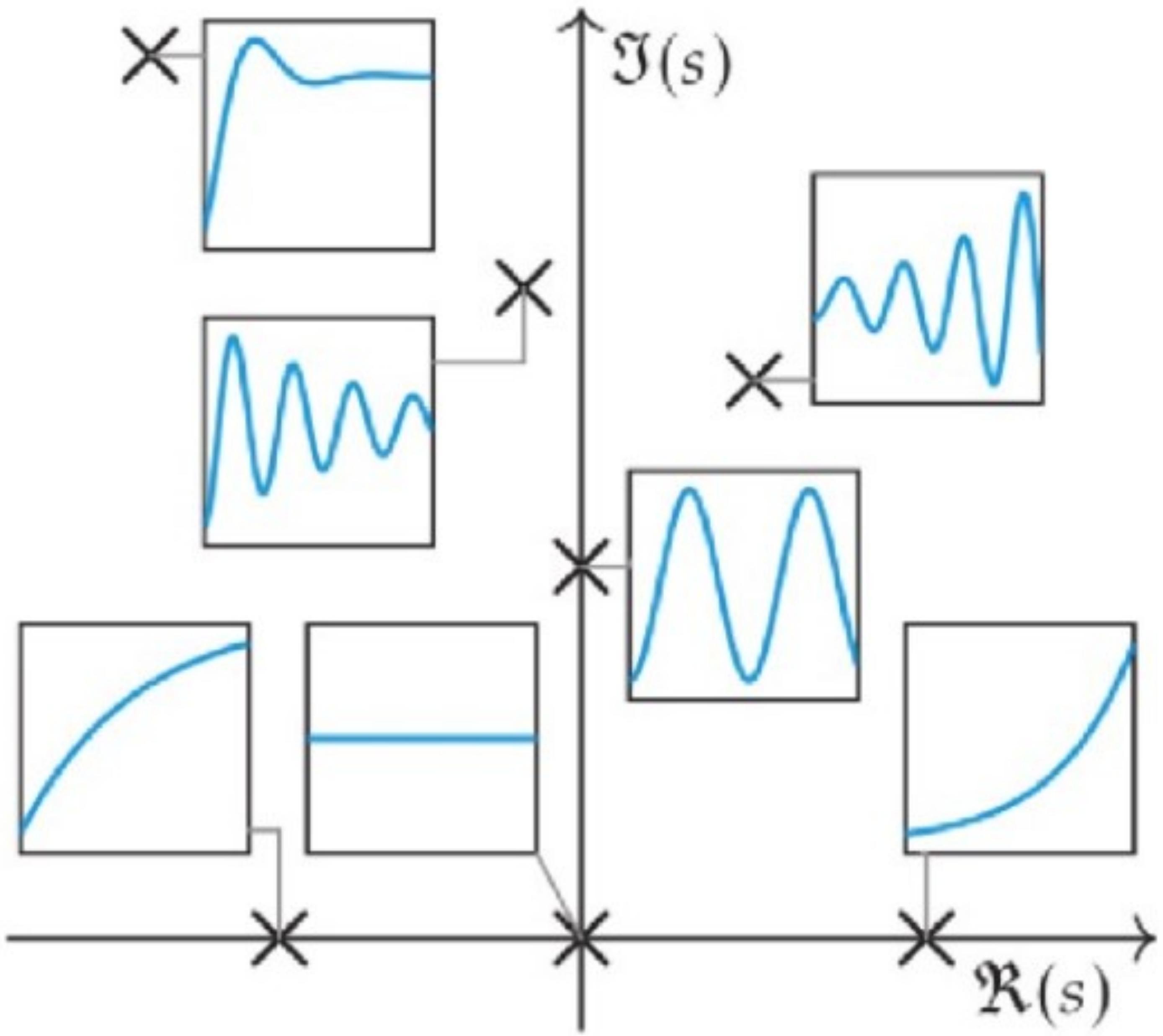

$$K_p = \lim_{s\to 0} G(s)$$

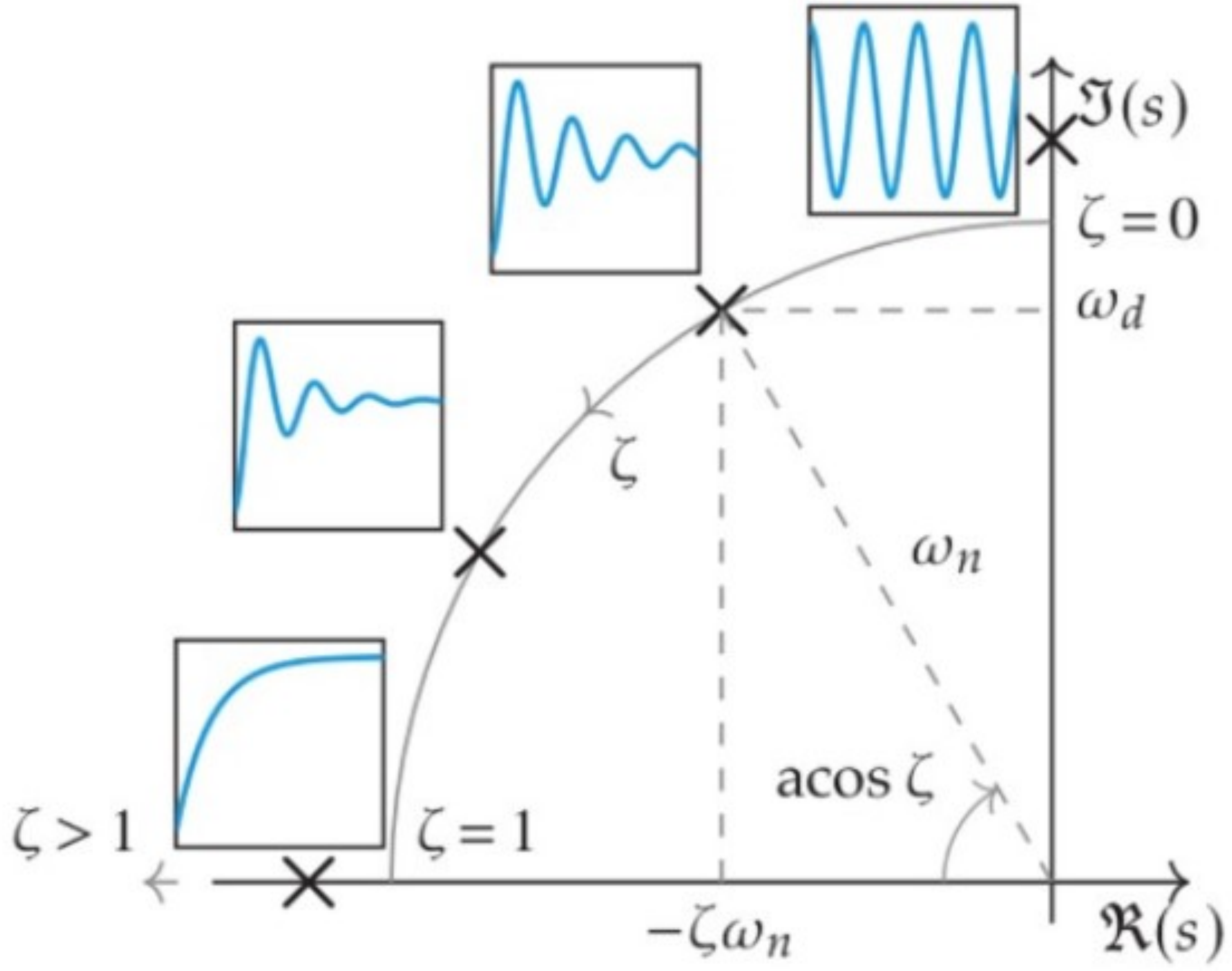
$$= \frac{450(8)(12)(15)}{0(38)(0+0+28)} = \infty$$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

$$K_v = \lim_{s\to 0} sG(s) = \frac{450(8)(12)(15)}{38(28)} = 609$$

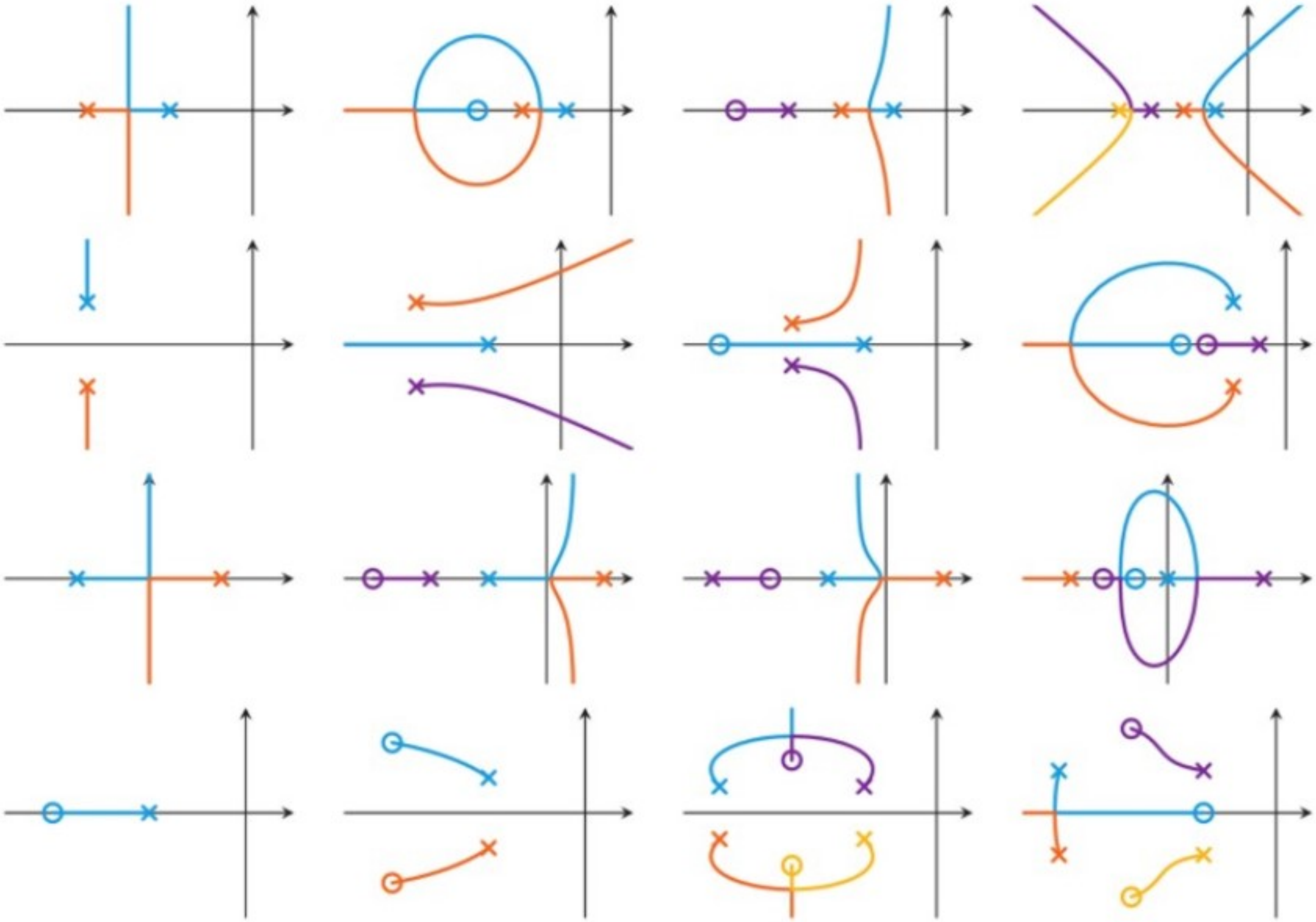
$$e_{ss} = 37\frac{1}{609} = 0.06$$

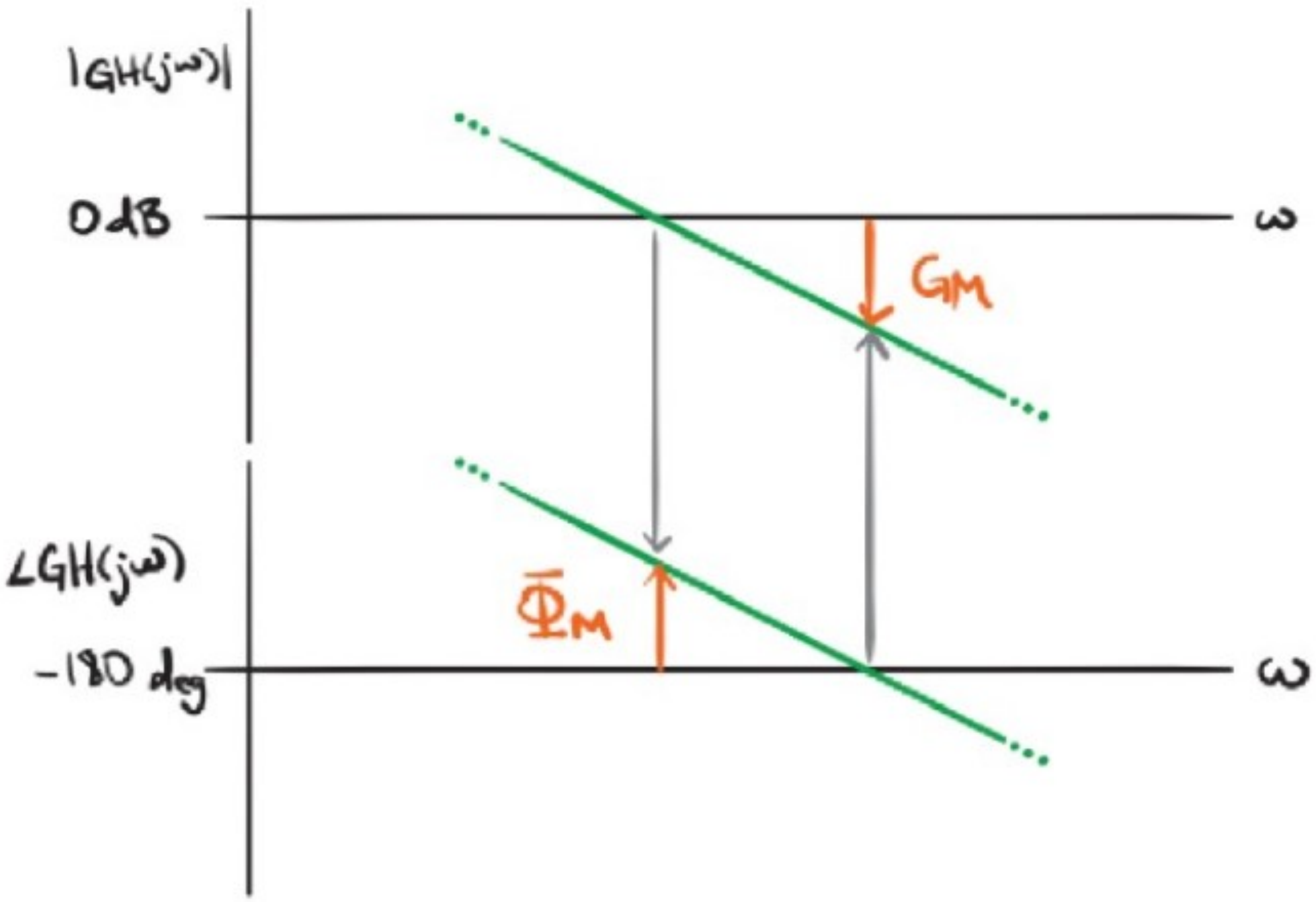



$$G_{CL}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

**Problem 7.1** 7X If a closed-loop system has a set of complex conjugate poles at $p = -2.5 \pm j4.33$ and no zeros, what is the system's natural frequency, damping ratio, overshoot, and settling time?





|GH(jω)|
0dB
ω
GM
∠GH(jω)
ΦM
-180 deg
ω

1. Tell how many roots of the following polynomial are in the right half-plane, in the left half-plane, and on the $j\omega$-axis: [Section: 6.2]

$$P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$$