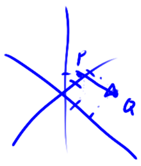
9-1

1–5 COMPONENTS AND LENGTH

Find the components of the vector **v** with initial point P and terminal point Q. Find $|\mathbf{v}|$. Sketch $|\mathbf{v}|$. Find the unit vector **u** in the direction of **v**.

1. P: (1, 1, 0), Q: (6, 2, 0)
2. P: (1, 1, 1), Q: (2, 2, 0)
3. P: (−3.0, 4,0, −0.5), Q: (5.5, 0, 1.2)
4. P: (1, 4, 2), Q: (−1, −4, −2)
5. P: (0, 0, 0), Q: (2, 1, −2)



$$V = Q - P = \begin{bmatrix}2\\2\\0\end{bmatrix} - \begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}$$

$$|V| = \left(\sum_{i=1}^{N} V_i^2\right)^{1/2} = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

$$u = \frac{V}{|V|} = \frac{V}{\sqrt{3}} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

11–18 ADDITION, SCALAR MULTIPLICATION

Let $\mathbf{a} = [3, 2, 0] = 3\mathbf{i} + 2\mathbf{j}$; $\mathbf{b} = [-4, 6, 0] = 4\mathbf{i} + 6\mathbf{j}$, $\mathbf{c} = [5, -1, 8] = 5\mathbf{i} - \mathbf{j} + 8\mathbf{k}$, $\mathbf{d} = [0, 0, 4] = 4\mathbf{k}$.
Find:

11. $2\mathbf{a}$, $\frac{1}{2}\mathbf{a}$, $-\mathbf{a}$
12. $(\mathbf{a} + \mathbf{b}) + \mathbf{c}$, $\mathbf{a} + (\mathbf{b} + \mathbf{c})$
13. $\mathbf{b} + \mathbf{c}$, $\mathbf{c} + \mathbf{b}$
14. $3\mathbf{c} - 6\mathbf{d}$, $3(\mathbf{c} - 2\mathbf{d})$
15. $7(\mathbf{c} - \mathbf{b})$, $7\mathbf{c} - 7\mathbf{b}$
16. $\frac{9}{2}\mathbf{a} - 3\mathbf{c}$, $9(\frac{1}{2}\mathbf{a} - \frac{1}{3}\mathbf{c})$
17. $(7 - 3)\mathbf{a}$, $7\mathbf{a} - 3\mathbf{a}$
18. $4\mathbf{a} + 3\mathbf{b}$, $-4\mathbf{a} - 3\mathbf{b}$
19. What laws do Probs. 12–16 illustrate?
20. Prove Eqs. (4) and (6).

$$7(c-b) = 7c - 7b$$

$$= 7\begin{bmatrix}5\\-1\\8\end{bmatrix} - 7\begin{bmatrix}-4\\6\\0\end{bmatrix} = \begin{bmatrix}35\\-7\\56\end{bmatrix} - \begin{bmatrix}-28\\42\\0\end{bmatrix} = \begin{bmatrix}35\\-7\\56\end{bmatrix} + \begin{bmatrix}28\\-42\\0\end{bmatrix} = \begin{bmatrix}63\\-49\\56\end{bmatrix}$$

$$7(c-b) = 7\left(\begin{bmatrix}5\\-1\\8\end{bmatrix} - \begin{bmatrix}-4\\6\\0\end{bmatrix}\right) = 7\begin{bmatrix}9\\-7\\8\end{bmatrix} = \begin{bmatrix}63\\-49\\56\end{bmatrix}$$

21–25 FORCES, RESULTANT

Find the resultant in terms of components and its magnitude.

21. $\mathbf{p} = [2, 3, 0]$, $\mathbf{q} = [0, 6, 1]$, $\mathbf{u} = [2, 0, -4]$
22. $\mathbf{p} = [1, -2, 3]$, $\mathbf{q} = [3, 21, -16]$, $\mathbf{u} = [-4, -19, 13]$
23. $\mathbf{u} = [8, -1, 0]$, $\mathbf{v} = [\frac{1}{2}, 0, \frac{4}{3}]$, $\mathbf{w} = [-\frac{17}{2}, 1, \frac{11}{3}]$
24. $\mathbf{p} = [-1, 2, -3]$, $\mathbf{q} = [1, 1, 1]$, $\mathbf{u} = [1, -2, 2]$
25. $\mathbf{u} = [3, 1, -6]$, $\mathbf{v} = [0, 2, 5]$, $\mathbf{w} = [3, -1, -13]$

$$p + q + u = \begin{bmatrix}2\\3\\0\end{bmatrix} + \begin{bmatrix}0\\6\\1\end{bmatrix} + \begin{bmatrix}2\\0\\-4\end{bmatrix} = \begin{bmatrix}4\\9\\-3\end{bmatrix}$$

$$|p+q+u| = \sqrt{4^2 + 9^2 + (-3)^2} = \sqrt{16 + 81 + 9} = \sqrt{106}$$

9-2

1–10 INNER PRODUCT

Let $\mathbf{a} = [1, -3, 5]$, $\mathbf{b} = [4, 0, 8]$, $\mathbf{c} = [-2, 9, 1]$.
Find:

1. $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{b} \cdot \mathbf{a}$, $\mathbf{b} \cdot \mathbf{c}$
2. $(-3\mathbf{a} + 5\mathbf{c}) \cdot \mathbf{b}$, $15(\mathbf{a} - \mathbf{c}) \cdot \mathbf{b}$
3. $|\mathbf{a}|$, $|2\mathbf{b}|$, $|-\mathbf{c}|$
4. $|\mathbf{a} + \mathbf{b}|$, $|\mathbf{a}| + |\mathbf{b}|$
5. $|\mathbf{b} + \mathbf{c}|$, $|\mathbf{b}| + |\mathbf{c}|$
6. $|\mathbf{a} + \mathbf{c}|^2 + |\mathbf{a} - \mathbf{c}|^2 - 2(|\mathbf{a}|^2 + |\mathbf{c}|^2)$
7. $|\mathbf{a} \cdot \mathbf{c}|$, $|\mathbf{a}||\mathbf{c}|$
8. $5\mathbf{a} \cdot 13\mathbf{b}$, $65\mathbf{a} \cdot \mathbf{b}$
9. $15\mathbf{a} \cdot \mathbf{b} + 15\mathbf{a} \cdot \mathbf{c}$, $15\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$
10. $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$, $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{c}$

$$5a \cdot 13b = 5\begin{bmatrix}1\\-3\\5\end{bmatrix} \cdot 13\begin{bmatrix}4\\0\\8\end{bmatrix} = \begin{bmatrix}5\\-15\\25\end{bmatrix} \cdot \begin{bmatrix}52\\0\\104\end{bmatrix} = 5(52) + (-15)0 + (25)104 = 2860$$

$$65\, a \cdot b = 65(a \cdot b) = 65\left(\begin{bmatrix}1\\-3\\5\end{bmatrix} \cdot \begin{bmatrix}4\\0\\8\end{bmatrix}\right) = 65(4 + 40) = 65(44) = 2860$$

17–20 WORK

Find the work done by a force **p** acting on a body if the body is displaced along the straight segment $\overline{AB}$ from A to B. Sketch $\overline{AB}$ and **p**. Show the details.

17. $\mathbf{p} = [2, 5, 0]$, A: (1, 3, 3), B: (3, 5, 5)
18. $\mathbf{p} = [-1, -2, 4]$, A: (0, 0, 0), B: (6, 7, 5)
19. $\mathbf{p} = [0, 4, 3]$, A: (4, 5, −1), B: (1, 3, 0)
20. $\mathbf{p} = [6, -3, -3]$, A: (1, 5, 2), B: (3, 4, 1)
21. **Resultant.** Is the work done by the resultant of two forces in a displacement the sum of the work done by each of the forces separately? Give proof or counterexample.

$W = F \cdot d$ $\quad \cancel{W = Fd}$

$$W = \begin{bmatrix}2\\5\\0\end{bmatrix} \cdot \left(\begin{bmatrix}3\\5\\5\end{bmatrix} - \begin{bmatrix}1\\3\\3\end{bmatrix}\right) = \begin{bmatrix}2\\5\\0\end{bmatrix} \cdot \begin{bmatrix}2\\2\\2\end{bmatrix} = 4 + 10 + 0 = 14$$

31–35 ORTHOGONALITY

ORTHOGONALITY is particularly important, mainly because of orthogonal coordinates, such as ***Cartesian coordinates***, whose ***natural basis*** [Eq. (9), Sec. 9.1], consists of three orthogonal unit vectors.

31. For what values of a_1 are $[a_1, 4, 3]$ and $[3, -2, 12]$ orthogonal?
32. **Planes.** For what c are $3x + z = 5$ and $8x - y + cz = 9$ orthogonal?
33. **Unit vectors.** Find all unit vectors $\mathbf{a} = [a_1, a_2]$ in the plane orthogonal to [4, 3].
34. **Corner reflector.** Find the angle between a light ray and its reflection in three orthogonal plane mirrors, known as *corner reflector*.
35. **Parallelogram.** When will the diagonals be orthogonal? Give a proof.

$$a \cdot b = |a||b|\cos(\theta)$$

$$\begin{bmatrix}a_1\\4\\3\end{bmatrix} \cdot \begin{bmatrix}3\\-2\\12\end{bmatrix} = 0$$

$$3a_1 - 8 + 36 = 0$$

$$3a_1 = 8 - 36$$

$$a_1 = \frac{8-36}{3} = \frac{-28}{3}$$

36–40 COMPONENT IN THE DIRECTION OF A VECTOR

Find the component of **a** in the direction of **b**. Make a sketch.

36. $\mathbf{a} = [1, 1, 1]$, $\mathbf{b} = [2, 1, 3]$
37. $\mathbf{a} = [3, 4, 0]$, $\mathbf{b} = [4, -3, 2]$
38. $\mathbf{a} = [8, 2, 0]$, $\mathbf{b} = [-4, -1, 0]$
39. When will the component (the projection) of **a** in the direction of **b** be equal to the component (the projection) of **b** in the direction of **a**? First guess.
40. What happens to the component of **a** in the direction of **b** if you change the length of **b**?

$$\frac{a \cdot b}{|b|^2}\, b$$

$$\left\|\begin{bmatrix}4\\-3\\2\end{bmatrix}\right\| = \sqrt{16 + 9 + 4} = \sqrt{29}$$

$$\frac{\begin{bmatrix}3\\4\\0\end{bmatrix} \cdot \begin{bmatrix}4\\-3\\2\end{bmatrix}}{\left\|\begin{bmatrix}4\\-3\\2\end{bmatrix}\right\|^2} \begin{bmatrix}4\\-3\\2\end{bmatrix} = \frac{12 - 12 + 0}{29}\begin{bmatrix}4\\-3\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} = \vec{0}$$

$a = [1, 1, 1]$ $\quad b = [2, 1, 3]$

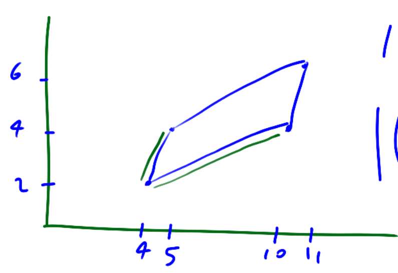
$$\frac{\begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}2\\1\\3\end{bmatrix}}{\left\|\begin{bmatrix}2\\1\\3\end{bmatrix}\right\|^2} \begin{bmatrix}2\\1\\3\end{bmatrix} = \frac{2 + 1 + 3}{14}\begin{bmatrix}2\\1\\3\end{bmatrix} = \frac{6}{14}\begin{bmatrix}2\\1\\3\end{bmatrix} = \begin{bmatrix}0.86\\0.43\\1.29\end{bmatrix}$$

$$\left\|\begin{bmatrix}2\\1\\3\end{bmatrix}\right\| = \sqrt{4 + 1 + 9} = \sqrt{14}$$

9-3

25–35 APPLICATIONS

25. **Moment m of a force p.** Find the moment vector **m** and m of $\mathbf{p} = [2, 3, 0]$ about Q: (2, 1, 0) acting on a line through A: (0, 3, 0). Make a sketch.
26. **Moment.** Solve Prob. 25 if $\mathbf{p} = [1, 0, 3]$, Q: (2, 0, 3), and A: (4, 3, 5).
27. **Parallelogram.** Find the area if the vertices are (4, 2, 0), (10, 4, 0), (5, 4, 0), and (11, 6, 0). Make a sketch.



$$|a \times b| = A$$

$$\left|\left(\begin{bmatrix}10\\4\\0\end{bmatrix} - \begin{bmatrix}4\\2\\0\end{bmatrix}\right) \times \left(\begin{bmatrix}5\\4\\0\end{bmatrix} - \begin{bmatrix}4\\2\\0\end{bmatrix}\right)\right| = \left|\begin{bmatrix}6\\2\\0\end{bmatrix} \times \begin{bmatrix}1\\2\\0\end{bmatrix}\right| = \left|\begin{bmatrix}0\\0\\10\end{bmatrix}\right| = 10$$

$$\begin{vmatrix} i & j & k \\ 6 & 2 & 0 \\ 1 & 2 & 0 \end{vmatrix} \begin{matrix} i & j \\ 6 & 2 \\ 1 & 2 \end{matrix} = \cancel{i(2)(0)} + \cancel{j(0)(1)} + k(6)(2) - k(2)(1) - \cancel{i(0)(2)} - \cancel{j(6)(0)}$$

$$= 12k - 2k = 10k = \begin{bmatrix}0\\0\\10\end{bmatrix}$$

9-4

1–8 SCALAR FIELDS IN THE PLANE

Let the temperature T in a body be independent of z so that it is given by a scalar function $T = T(x, t)$. Identify the isotherms $T(x, y) = \text{const}$. Sketch some of them.

1. $T = x^2 - y^2$
2. $T = xy$
3. $T = 3x - 4y$
4. $T = \arctan(y/x)$
5. $T = y/(x^2 + y^2)$
6. $T = x/(x^2 + y^2)$
7. $T = 9x^2 + 4y^2$

$$\frac{x}{x^2 + y^2} = c$$

$$x = c(x^2 + y^2)$$

$$\frac{x}{c} = x^2 + y^2$$

$$x^2 - \frac{x}{c} + y^2 = 0$$

$$x^2 - \frac{x}{c} + \frac{1}{4c^2} + y^2 = \frac{1}{4c^2} \longrightarrow \left(x - \frac{1}{2c}\right)^2 + y^2 = \frac{1}{4c^2}$$

$$\left(x - \frac{1}{2c}\right)^2 = x^2 - \frac{x}{c} + \frac{1}{4c^2}$$

if $c = \frac{1}{2}$ $\quad (x - 1)^2 + y^2 = 1$