reals

$\mathbb{R}$

1.2 π e

integers

$\mathbb{Z}$

1 5 26 -3

complex

$\mathbb{C}$

$3+5i$ $\quad -6+\pi i$

$\mathbb{R}^3 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| x, y, z \in \mathbb{R} \right\}$

$\mathbb{R}^n$ n-dimensional vector of reals

$\mathbb{Z}^n$ n-dimensional vector of integers

$\mathbb{C}^n$ n-dimensional vector of complex numbers

$\begin{bmatrix} 3 \\ -2.5 \\ 6 \end{bmatrix} \in \mathbb{R}^3$ $\qquad \vec{v} \in \mathbb{R}^4$

sets.exe Exercises for Chapter sets

Exercise sets.hardhat

For the following, write the set described in set-builder notation.

a. $A = \{2, 3, 5, 9, 17, 33, \cdots\}$.
b. B is the set of integers divisible by 11.
c. $C = \{1/3, 1/4, 1/5, \cdots\}$.
d. D is the set of reals between -3 and 42.

$A - 1 = \{1, 2, 4, 8, 16, 32, \ldots\}$

$A = \{2^n + 1 \mid n \in \mathbb{Z}, n \geq 0\} = \{2, 3, 5, 9, \ldots\}$

$B = \{11n \mid n \in \mathbb{Z}\}$ $\qquad B = \{n \mid \frac{n}{11} \in \mathbb{Z}\}$

$C = \left\{\frac{1}{n} \mid n \in \mathbb{Z}, n \geq 3\right\}$

$D = \{x \mid x \in \mathbb{R}, x > -3, x < 42\}$

Exercise sets.2

Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. Prove the Cauchy-Schwarz Inequality

$$|\mathbf{x} \cdot \mathbf{y}| \leqslant \|\mathbf{x}\| \|\mathbf{y}\|. \tag{1}$$

Hint: you may find the geometric definition of the dot product helpful.

$x \cdot y = \|x\| \|y\| \cos(\theta)$

$\cos(\theta) \leq 1$

$|x \cdot y| \leq \|x\| \|y\|$

Exercise sets.3

Let $\mathbf{x} \in \mathbb{R}^n$. Prove that

$$\mathbf{x} \cdot \mathbf{x} = \|\mathbf{x}\|^2. \tag{2}$$

Hint: you may find the geometric definition of the dot product helpful.

$x \cdot x = \|x\| \|x\| \cos(\theta) = \|x\| \|x\| \cos(0) = \|x\| \|x\| = \|x\|^2$

$x \cdot x = \|x\|^2$

Exercise sets.4

Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. Prove the Triangle Inequality

$$\|\mathbf{x} + \mathbf{y}\| \leqslant \|\mathbf{x}\| + \|\mathbf{y}\|. \tag{3}$$

Hint: you may find the Cauchy-Schwarz Inequality helpful.

Ass. 1

$\|x + y\|^2 \leq (\|x\| + \|y\|)^2$

$(\|x\| + \|y\|)^2 = \|x\|^2 + 2\|x\|\|y\| + \|y\|^2$

$\|x + y\|^2 \leq \|x\|^2 + 2\|x\|\|y\| + \|y\|^2$

prob Probability