

# 11-01\_Expectation\_and\_Variance

November 1, 2021

```
[1]: import numpy as np
import matplotlib.pyplot as plt
import json
%matplotlib inline
```

```
[2]: !cat 11-01_compiled.json
```

```
{
  "volume_removed": 4.241592920353988e-06, "work": 3347.810682302913},
  {"volume_removed": 2.698230088495556e-06, "work": 1953.680992480751},
  {"volume_removed": 2.300884955752207e-06, "work": 1363.2789677561075},
  {"volume_removed": 2.452212389380519e-06, "work": 1819.9940807227508},
  {"volume_removed": 1.9991150442478003e-06, "work": 1037.0236290264234},
  {"volume_removed": 2.8946902654867138e-06, "work": 1741.4358847637343},
  {"volume_removed": 3.4327433628318754e-06, "work": 3059.448180992309},
  {"volume_removed": 4.536283185840687e-06, "work": 3395.929654030393},
  {"volume_removed": 2.9690265486725823e-06, "work": 2143.2377373273703},
  {"volume_removed": 2.4761061946902418e-06, "work": 2250.2180182280063},
  {"volume_removed": 2.303539823008857e-06, "work": 1236.4107569338053},
  {"volume_removed": 2.4451327433628363e-06, "work": 1272.2595656171782},
  {"volume_removed": 4.333628318584063e-06, "work": 2717.600611010963},
  {"volume_removed": 2.7336283185840694e-06, "work": 1907.271837633588},
  {"volume_removed": 3.11238938053097e-06, "work": 2004.4611481017607},
  {"volume_removed": 5.799115044247785e-06, "work": 4270.038245625621},
  {"volume_removed": 3.407079646017694e-06, "work": 2232.7551425716583},
  {"volume_removed": 2.6345132743362865e-06, "work": 1420.0197699872383},
  {"volume_removed": 3.049557522123892e-06, "work": 1880.2359347697763},
  {"volume_removed": 5.258407079646024e-06, "work": 3621.6803961686687},
  {"volume_removed": 4.8610619469026506e-06, "work": 3054.87841204957},
  {"volume_removed": 2.5070796460176977e-06, "work": 1623.9679311977454},
  {"volume_removed": 2.9690265486725573e-06, "work": 2567.2327870069685},
  {"volume_removed": 4.153097345132754e-06, "work": 2840.663432431038},
  {"volume_removed": 3.88672566371681e-06, "work": 3245.6230117714035},
  {"volume_removed": 3.1017699115044207e-06, "work": 2167.55084221368},
  {"volume_removed": 4.8893805309734565e-06, "work": 3453.7549587100293},
  {"volume_removed": 5.299115044247787e-06, "work": 4109.662922568263},
  {"volume_removed": 2.283185840707976e-06, "work": 1667.3140039098253},
  {"volume_removed": 1.510619469026543e-06, "work": 1752.5632082607526},
  {"volume_removed": 5.303539823008845e-06, "work": 3675.635662475508},
}
```

```
{"volume_removed": 4.1283185840707895e-06, "work": 2653.3471718872415}]
```

```
[3]: with open("11-01_compiled.json") as f:  
      data = json.load(f)
```

```
[4]: data
```

```
[4]: [{'volume_removed': 4.241592920353988e-06, 'work': 3347.810682302913},  
      {'volume_removed': 2.698230088495556e-06, 'work': 1953.680992480751},  
      {'volume_removed': 2.300884955752207e-06, 'work': 1363.2789677561075},  
      {'volume_removed': 2.452212389380519e-06, 'work': 1819.9940807227508},  
      {'volume_removed': 1.9991150442478003e-06, 'work': 1037.0236290264234},  
      {'volume_removed': 2.8946902654867138e-06, 'work': 1741.4358847637343},  
      {'volume_removed': 3.4327433628318754e-06, 'work': 3059.448180992309},  
      {'volume_removed': 4.536283185840687e-06, 'work': 3395.929654030393},  
      {'volume_removed': 2.9690265486725823e-06, 'work': 2143.2377373273703},  
      {'volume_removed': 2.4761061946902418e-06, 'work': 2250.2180182280063},  
      {'volume_removed': 2.303539823008857e-06, 'work': 1236.4107569338053},  
      {'volume_removed': 2.4451327433628363e-06, 'work': 1272.2595656171782},  
      {'volume_removed': 4.333628318584063e-06, 'work': 2717.600611010963},  
      {'volume_removed': 2.7336283185840694e-06, 'work': 1907.271837633588},  
      {'volume_removed': 3.11238938053097e-06, 'work': 2004.4611481017607},  
      {'volume_removed': 5.799115044247785e-06, 'work': 4270.038245625621},  
      {'volume_removed': 3.407079646017694e-06, 'work': 2232.7551425716583},  
      {'volume_removed': 2.6345132743362865e-06, 'work': 1420.0197699872383},  
      {'volume_removed': 3.049557522123892e-06, 'work': 1880.2359347697763},  
      {'volume_removed': 5.258407079646024e-06, 'work': 3621.6803961686687},  
      {'volume_removed': 4.8610619469026506e-06, 'work': 3054.87841204957},  
      {'volume_removed': 2.5070796460176977e-06, 'work': 1623.9679311977454},  
      {'volume_removed': 2.9690265486725573e-06, 'work': 2567.2327870069685},  
      {'volume_removed': 4.153097345132754e-06, 'work': 2840.663432431038},  
      {'volume_removed': 3.88672566371681e-06, 'work': 3245.6230117714035},  
      {'volume_removed': 3.1017699115044207e-06, 'work': 2167.55084221368},  
      {'volume_removed': 4.8893805309734565e-06, 'work': 3453.7549587100293},  
      {'volume_removed': 5.299115044247787e-06, 'work': 4109.662922568263},  
      {'volume_removed': 2.283185840707976e-06, 'work': 1667.3140039098253},  
      {'volume_removed': 1.510619469026543e-06, 'work': 1752.5632082607526},  
      {'volume_removed': 5.303539823008845e-06, 'work': 3675.635662475508},  
      {'volume_removed': 4.1283185840707895e-06, 'work': 2653.3471718872415}]
```

```
[5]: type(data)
```

```
[5]: list
```

```
[6]: n = len(data)  
      volume_removed = np.zeros(n)  
      work = np.zeros_like(volume_removed)  
      for i, d in enumerate(data):
```

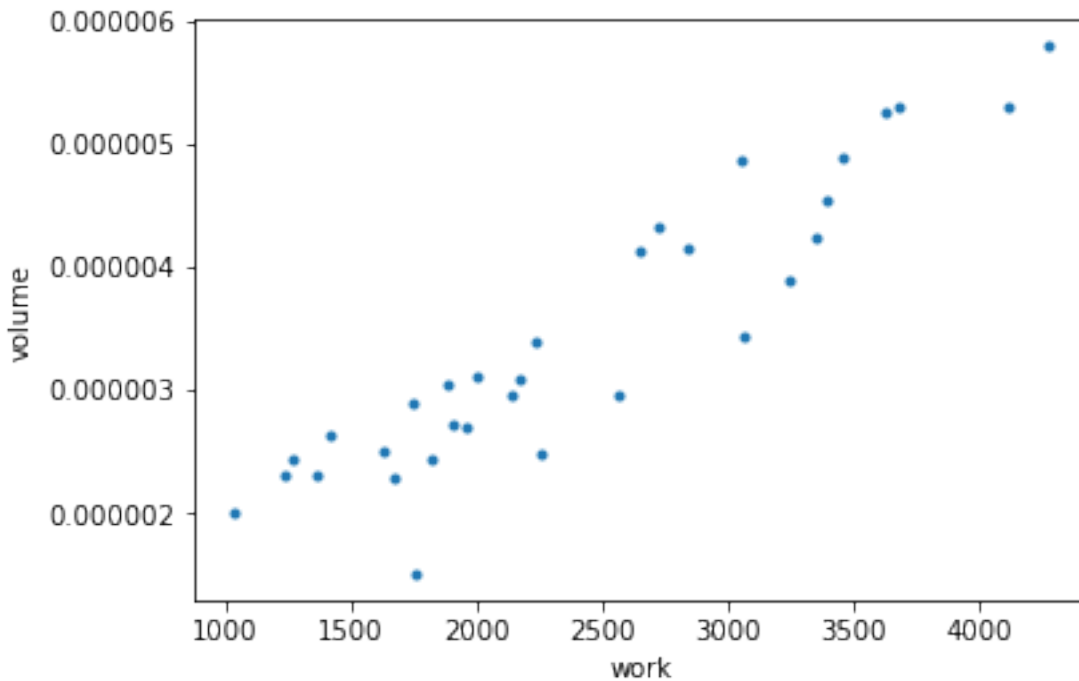
```
volume_removed[i] = d['volume_removed']
work[i] = d['work']
```

```
[7]: work
```

```
[7]: array([3347.8106823 , 1953.68099248, 1363.27896776, 1819.99408072,
        1037.02362903, 1741.43588476, 3059.44818099, 3395.92965403,
        2143.23773733, 2250.21801823, 1236.41075693, 1272.25956562,
        2717.60061101, 1907.27183763, 2004.4611481 , 4270.03824563,
        2232.75514257, 1420.01976999, 1880.23593477, 3621.68039617,
        3054.87841205, 1623.9679312 , 2567.23278701, 2840.66343243,
        3245.62301177, 2167.55084221, 3453.75495871, 4109.66292257,
        1667.31400391, 1752.56320826, 3675.63566248, 2653.34717189])
```

```
[8]: plt.plot(work, volume_removed, '.')
plt.xlabel('work')
plt.ylabel('volume')
```

```
[8]: Text(0, 0.5, 'volume')
```



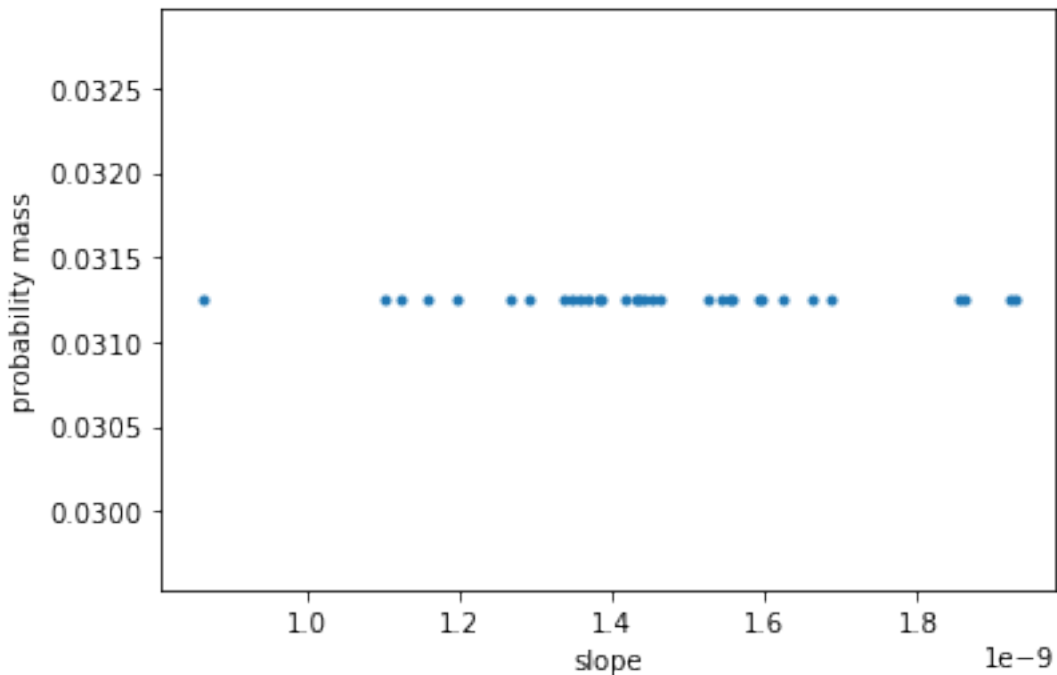
```
[9]: slope = volume_removed / work
probability_mass = np.ones_like(slope) / slope.size
```

```
[10]: np.sum(probability_mass)
```

```
[10]: 1.0
```

```
[11]: plt.plot(slope, probability_mass, '.')  
plt.xlabel('slope')  
plt.ylabel('probability mass')
```

```
[11]: Text(0, 0.5, 'probability mass')
```



```
[12]: expectation = np.sum(slope * probability_mass)  
print(expectation)
```

```
1.459773700405994e-09
```

```
[13]: variance = np.sum((slope - expectation)**2 * probability_mass)  
print(variance)
```

```
5.704095919684368e-20
```

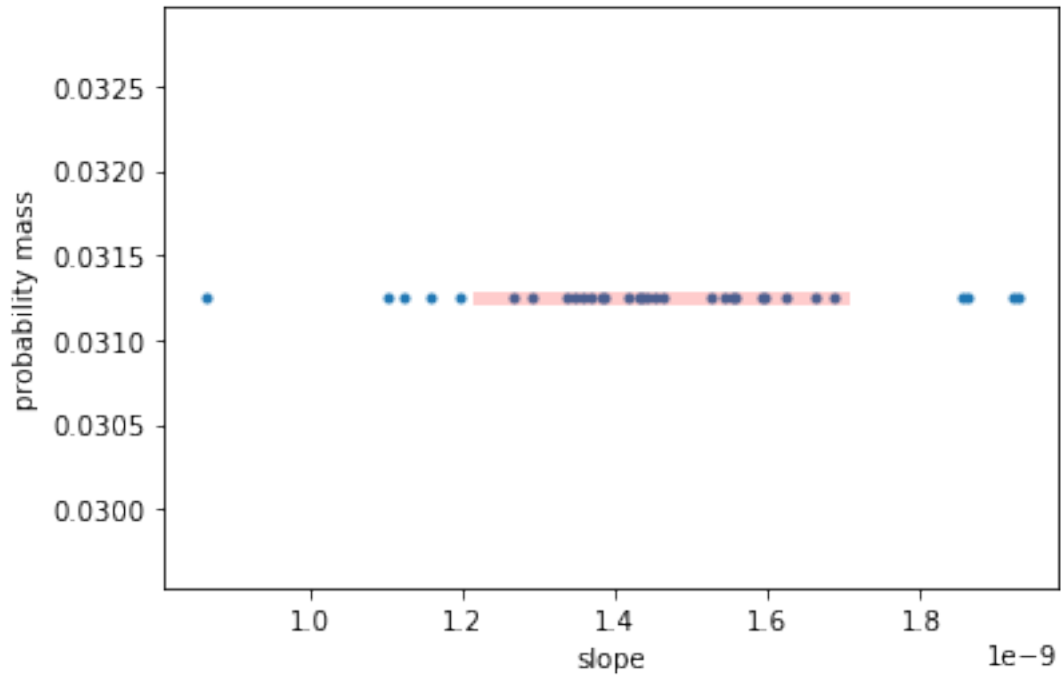
```
[14]: std_dev = np.sqrt(variance)  
print(std_dev)
```

```
2.388324919202655e-10
```

```
[15]: plt.plot(slope, probability_mass, '.')  
plt.plot([expectation - std_dev, expectation + std_dev], probability_mass[:2],  
        ↪ 'r', alpha=0.2, linewidth=5)
```

```
plt.xlabel('slope')
plt.ylabel('probability mass')
```

```
[15]: Text(0, 0.5, 'probability mass')
```



```
[16]: type(work)
```

```
[16]: numpy.ndarray
```

```
[17]: a = 1
b = 1.5
c = 'test'
print(type(a))
print(type(b))
print(type(c))
```

```
<class 'int'>
<class 'float'>
<class 'str'>
```

```
[ ]:
```