11-05_Kr_Problem_9.5.41

November 5, 2021

[3]:
```
from sympy import *
```

[4]:
```
var
```

[4]:
```
<function sympy.core.symbol.var(names, **args)>
```

[5]:
```
var('t')
```

[5]:
$$t$$

[6]:
```
r = Matrix([2 * cos(t) + cos(2 * t), 2 * sin(t) - sin(2 * t)])
```

[7]:
```
r
```

[7]:
$$\begin{bmatrix} 2\cos(t) + \cos(2t) \\ 2\sin(t) - \sin(2t) \end{bmatrix}$$

[9]:
```
v = diff(r, t)
v
```

[9]:
$$\begin{bmatrix} -2\sin(t) - 2\sin(2t) \\ 2\cos(t) - 2\cos(2t) \end{bmatrix}$$

$$\sqrt{v \cdot v}$$

[14]:
```
speed = sqrt(v.dot(v))
speed
```

[14]:
$$\sqrt{(-2\sin(t) - 2\sin(2t))^2 + (2\cos(t) - 2\cos(2t))^2}$$

[17]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

[18]:
```
t = np.arange(0, 4 * np.pi, 0.05)
```

[27]:
```
r_numeric = np.zeros([t.size, 2])
for i, ti in enumerate(t):
  r_numeric[i,:] = np.array(r.subs({'t': ti}).evalf().tolist()).flatten()
```

[28]:
```
r_numeric
```

[28]:
```
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 [ 2.98679937e+00, -2.92044605e-04],
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```
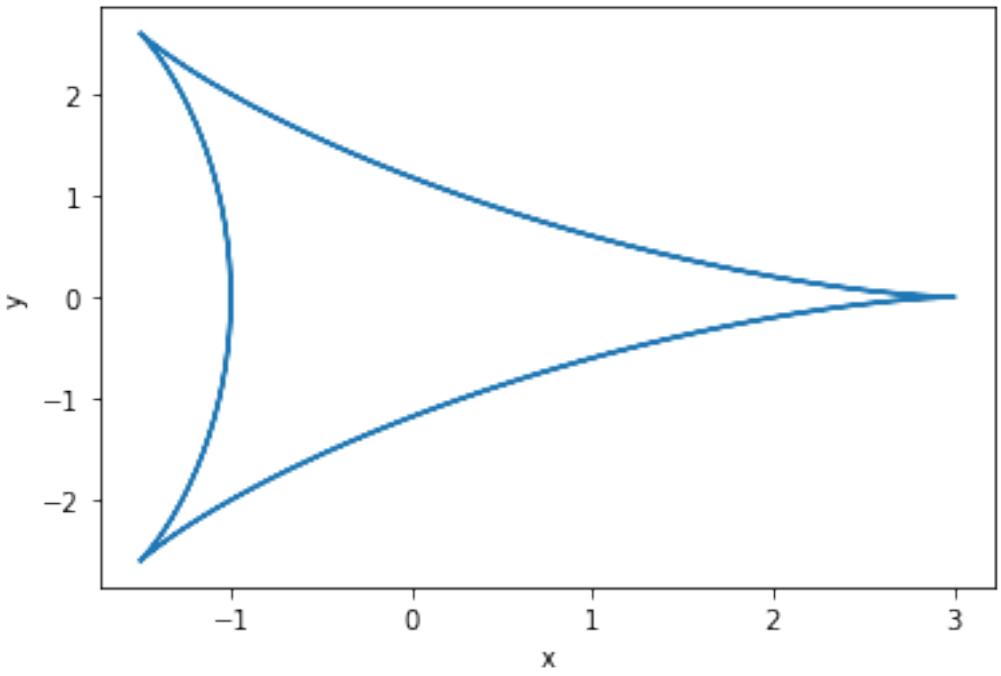
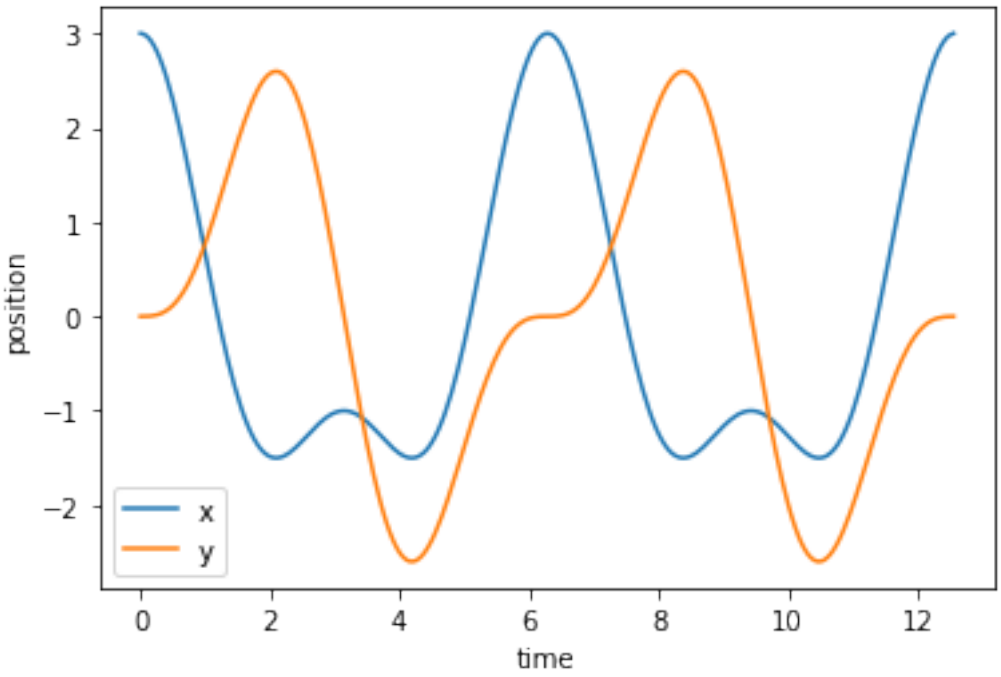

[32]:
```
plt.plot(r_numeric[:,0], r_numeric[:,1])
plt.xlabel('x')
plt.ylabel('y')
```

[32]:
```
Text(0, 0.5, 'y')
```



[35]:
```
plt.plot(t, r_numeric[:,0], label='x')
plt.plot(t, r_numeric[:,1], label='y')
plt.legend()
plt.xlabel('time')
plt.ylabel('position')
```

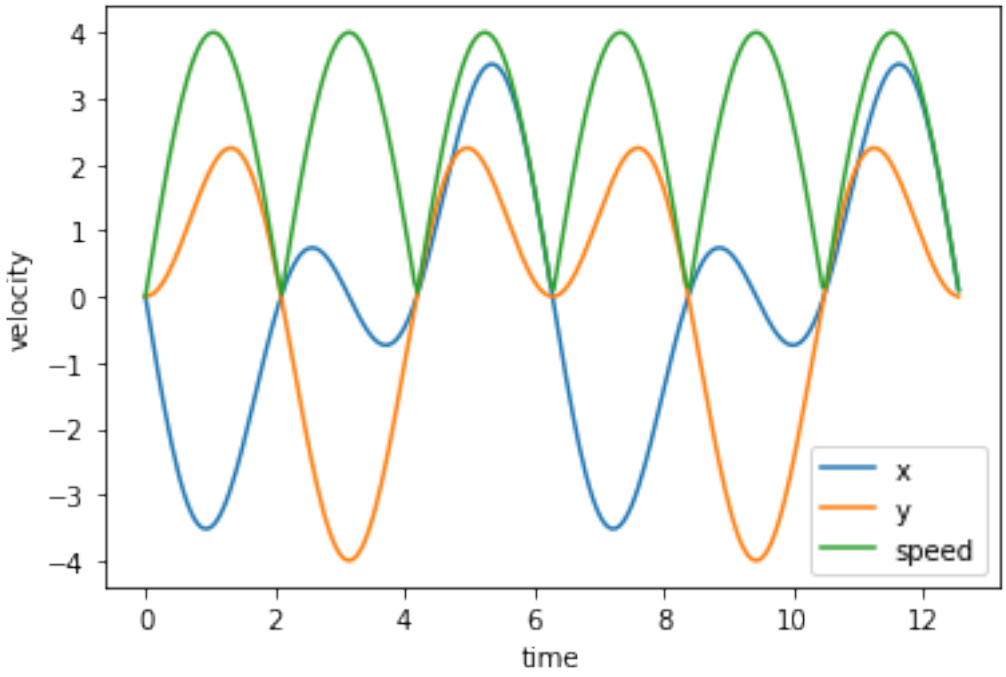
[35]: Text(0, 0.5, 'position')



[36]:
```
v_numeric = np.zeros([t.size, 2])
speed_numeric = np.zeros_like(t)
for i, ti in enumerate(t):
  v_numeric[i,:] = np.array(v.subs({'t': ti}).evalf().tolist()).flatten()
  speed_numeric[i] = speed.subs({'t': ti}).evalf()
```

[37]:
```
plt.plot(t, v_numeric[:,0], label='x')
plt.plot(t, v_numeric[:,1], label='y')
plt.plot(t, speed_numeric, label='speed')
plt.legend()
plt.xlabel('time')
plt.ylabel('velocity')
```

[37]: Text(0, 0.5, 'velocity')


velocity
time
x
y
speed

[ ]: