9.8.1

Find div v and its value at P

$v = [x^2, 4y^2, 9z^2] \qquad P = [-1, 0, \frac{1}{2}]$

$\text{div } v = \partial_x x^2 + \partial_y 4y^2 + \partial_z 9z^2$

$= 2x + 8y + 18z$

$V(x,y,z) = [x^2, 4y^2, 9z^2]$

$\text{div } v \ (x,y,z) = 2(-1) + 8(0) + 18(\frac{1}{2})$

$= -2 + 0 + 9 = 7$

9.8.5

$v = x^2y^2z^2[x,y,z] \qquad P = [3, -1, 4]$

$v(x,y,z) = [x^3y^2z^2, x^2y^3z^2, x^2y^2z^3]$

$\text{div } v = \partial_x x^3y^2z^2 + \partial_y x^2y^3z^2 + \partial_z x^2y^2z^3$

$= 3x^2y^2z^2 + 3x^2y^2z^2 + 3x^2y^2z^2$

$= 9x^2y^2z^2$

div v at $P = [3, -1, 4]$

$\text{div } v = 9(3)^2(-1)^2(4)^2 = 9(9)(1)(16) = 1296$

9.8.9

Prove $\text{div}(kv) = k \text{ div } v \qquad k \in \mathbb{R} \qquad v(\mathbb{R}^n) \to \mathbb{R}^n$

$\text{div}(kv) = \partial_1 kv_1 + \partial_2 kv_2 + \dots + \partial_n kv_n$

$= k\partial_1 v_1 + k\partial_2 v_2 + \dots + k\partial_n v_n$

$= k(\partial_1 v_1 + \partial_2 v_2 + \dots + \partial_n v_n)$

$= k \text{ div } v$ ✓

Prove $\text{div}(fv) = f \text{div } v + v \cdot \nabla f$

gradient