9.9.5

Find curl v

$v = xyz[x, y, z] = [x^2yz, \ xy^2z, xyz^2]$

$$\text{curl } v = [\partial_y v_z - \partial_z v_y, \ \partial_z v_x - \partial_x v_z, \ \partial_x v_y - \partial_y v_x]^T$$

$$= \begin{bmatrix} \partial_y \, xyz^2 - \partial_z \, xy^2z \\ \partial_z \, x^2yz - \partial_x \, xyz^2 \\ \partial_x \, xy^2z - \partial_y \, x^2yz \end{bmatrix} = \begin{bmatrix} xz^2 - xy^2 \\ x^2y - yz^2 \\ y^2z - x^2z \end{bmatrix} = \begin{bmatrix} x(z^2-y^2) \\ y(x^2-z^2) \\ z(y^2-x^2) \end{bmatrix}$$

9.9.14

show curl $(u+v)$ = curl (u) + curl(v)

$$\text{curl}(u+v) = \begin{bmatrix} \partial_y (u_z+v_z) - \partial_z (u_y+v_y) \\ \partial_z (u_x+v_x) - \partial_x (u_z+v_z) \\ \partial_x (u_y+v_y) - \partial_y (u_x-v_x) \end{bmatrix}$$

$$= \begin{bmatrix} \partial_y u_z + \partial_y v_z - \partial_z u_y - \partial_z v_y \\ \partial_z u_x + \partial_z v_x - \partial_x u_z - \partial_x v_z \\ \partial_x u_y + \partial_x v_y - \partial_y u_x - \partial_y v_x \end{bmatrix}$$

$$= \begin{bmatrix} \partial_y u_z - \partial_z u_y \\ \partial_z u_x - \partial_x u_z \\ \partial_x u_y - \partial_y u_x \end{bmatrix} + \begin{bmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{bmatrix}$$

$= \text{curl}(u) + \text{curl}(v)$ ✓

div $(\text{curl}(v)) = 0$

$$\text{div}(\text{curl}(v)) = \partial_x \, \text{curl}(v)_x + \partial_y \, \text{curl}(v)_y + \partial_z \, \text{curl}(v)_z$$

$$= \partial_x (\partial_y v_z - \partial_z v_y) + \partial_y (\partial_z v_x - \partial_x v_z)$$
$$+ \partial_z (\partial_x v_y - \partial_y v_x)$$

$$= \cancel{\partial_x \partial_y v_z} - \cancel{\partial_x \partial_z v_y} + \cancel{\partial_y \partial_z v_x} - \cancel{\partial_y \partial_x v_z}$$
$$+ \cancel{\partial_z \partial_x v_y} - \cancel{\partial_z \partial_y v_x}$$

Remember $\partial_x \partial_y = \partial_y \partial_x$

$= 0$ ✓

div $(u \times v) = v \cdot \text{curl } u - u \cdot \text{curl } v$

$$\text{div}(u \times v) = \text{div}\left(\begin{vmatrix} i & j & k \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} \right)$$

$$= \text{div}\,(i u_y v_z - i u_z v_y + j u_z v_x - j u_x v_z$$
$$+ k u_x v_y - k u_y v_x)$$

$$= \text{div}\left(\begin{bmatrix} u_y v_z - u_z v_y \\ u_z v_x - u_x v_z \\ u_x v_y - u_y v_x \end{bmatrix} \right)$$

$$= \partial_x (u_y v_z - u_z v_y) + \partial_y (u_z v_x - u_x v_z)$$
$$+ \partial_z (u_x v_y - u_y v_x)$$

$$v \cdot \text{curl } u = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \cdot \begin{bmatrix} \partial_y u_z - \partial_z u_y \\ \partial_z u_x - \partial_x u_z \\ \partial_x u_y - \partial_y u_x \end{bmatrix} = v_x (\partial_y u_z - \partial_z u_y)$$
$$+ v_y (\partial_z u_x - \partial_x u_z)$$
$$+ v_z (\partial_x u_y - \partial_y u_x)$$

$$u \cdot \text{curl } v = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \cdot \begin{bmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{bmatrix} = u_x (\partial_y v_z - \partial_z v_y)$$
$$+ u_y (\partial_z v_x - \partial_x v_z)$$
$$+ u_z (\partial_x v_y - \partial_y v_x)$$