Line Integral

$$F = \left[(xy)^2, \frac{2x^3y}{3}, z \right]$$

$r(t) = [t, t/2, t^2]$ from $t=0$ to $t=1$

$$W = \int_0^1 F(r(t)) \cdot r'(t)\, dt$$

$$r'(t) = [1, 1/2, 2t]$$

$$F(r(t)) = \left[\left(\frac{t^2}{2}\right)^2, \frac{\cancel{2}t^3\, t/\cancel{2}}{3}, t^2 \right]$$

$$= \left[\frac{t^4}{4}, \frac{t^4}{3}, t^2 \right]$$

$$F(r(t)) \cdot r'(t) = \left[\frac{t^4}{4}, \frac{t^4}{3}, t^2 \right] \cdot [1, 1/2, 2t]$$

$$= \frac{t^4}{4} + \frac{t^4}{6} + 2t^3$$

$$= \frac{3t^4}{12} + \frac{2t^4}{12} + 2t^3 = \frac{5t^4}{12} + 2t^3$$

$$\int_0^1 \frac{5t^4}{12} + 2t^3\, dt = \frac{5t^5}{5(12)} + \frac{2t^4}{4}\Big|_0^1 = \frac{\cancel{5}(1)}{\cancel{5}(12)} + \frac{2(1)}{4}$$

$$= \frac{1}{12} + \frac{1}{2} = \frac{1}{12} + \frac{6}{12}$$

$$\boxed{W = \frac{7}{12}} = 0.583$$

$$F = \left[(xy)^2, \frac{2x^3y}{3}, z \right]$$

$$\mathrm{Curl}(F) = \begin{bmatrix} \partial_y F_z - \partial_z F_y \\ \partial_z F_x - \partial_x F_z \\ \partial_x F_y - \partial_y F_x \end{bmatrix} = \begin{bmatrix} 0 - 0 \\ 0 - 0 \\ \frac{3(2)x^2y}{3} - 2x^2y \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 2x^2y - 2x^2y \end{bmatrix} = \vec{0}$$

$r(t) = [t, t/2, t^2]$ from $t=0$ to $t=1$

$r(0) = [0, 0, 0]$

$r(1) = [1, 1/2, 1]$

$r(t) = [t^2, t/2, t]$ from $t=0$ to $t=1$

$r'(t) = [2t, 1/2, 1]$

$$F(r(t)) = \left[(t^3\, t/2)^2, \frac{2(t^2)^3\, t/2}{3}, t \right]$$

$$= \left[\frac{t^8}{2}, \frac{t^7}{3}, t \right]$$

$$F(r(t)) \cdot r'(t) = \begin{bmatrix} t^8/2 \\ t^7/3 \\ t \end{bmatrix} \cdot \begin{bmatrix} 2t \\ 1/2 \\ 1 \end{bmatrix} = \frac{t^8}{2} 2t + \frac{t^7}{3} \frac{1}{2} + t$$

$$= t^9 + \frac{t^7}{6} + t$$

$$\int_0^1 t^9 + \frac{t^7}{6} + t\, dt = \frac{t^{10}}{10} + \frac{t^8}{8 \cdot 6} + \frac{t^2}{2}\Big|_0^1 = \frac{1}{10} + \frac{1}{48} + \frac{1}{2}$$

$$= 0.502$$