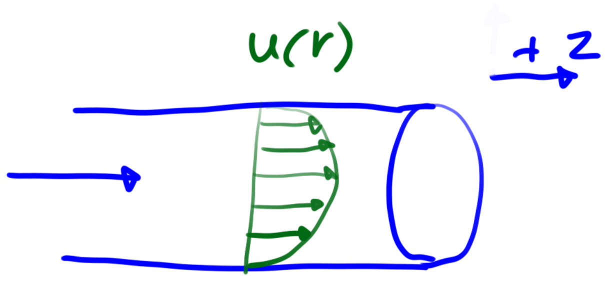

S is flat end of pipe

therefore $n = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\iint_S \vec{u}(r) \cdot n \, ds \qquad \vec{u}(r) = \begin{bmatrix} 0 \\ 0 \\ u(r) \end{bmatrix} \qquad u(r) = U\left(1 - \left(\frac{r}{R}\right)^2\right)$$

U is max vel

$$\vec{u}(r) \cdot n = \begin{bmatrix} 0 \\ 0 \\ u(r) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = u(r)$$

$$\iint_A U\left(1 - \left(\frac{r}{R}\right)^2\right) dA = \int_0^{2\pi}\int_0^R U\left(1 - \left(\frac{r}{R}\right)^2\right) r \, dr \, d\theta$$

$$= U \int_0^{2\pi}\int_0^R r - \frac{r^3}{R^2} \, dr \, d\theta = U \int_0^{2\pi} \frac{r^2}{2} - \frac{r^4}{4R^2}\bigg|_0^R d\theta$$

$$= U \int_0^{2\pi} \frac{R^2}{2} - \frac{R^4}{4R^2} \, d\theta = U \int_0^{2\pi} \frac{R^2}{2} - \frac{R^2}{4} \, d\theta$$

$$= U \int_0^{2\pi} \frac{R^2}{4} \, d\theta = \frac{U}{4} R^2 \theta \bigg|_0^{2\pi} = \frac{U R^2 2\pi}{4} = \frac{U R^2 \pi}{2}$$