11-10_Fourier_Series

November 11, 2021

[1]:

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

[4]:

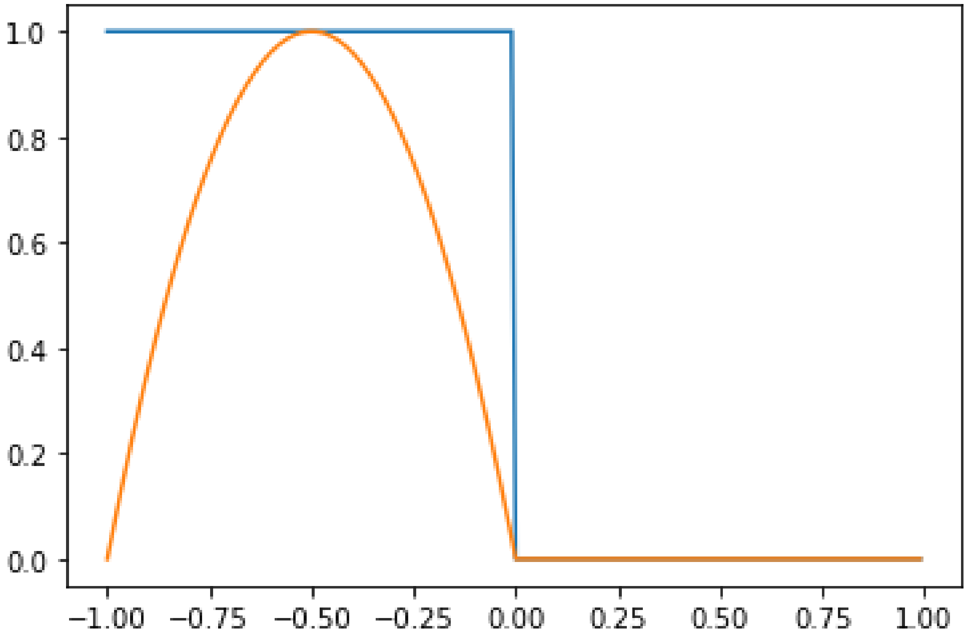
```
T = 2
dt = 0.01
t = np.arange(-T / 2, T / 2, dt)
```

[12]:

```
parabolic_mask = t < 0
parabolic = parabolic_mask * (1 - (2 * t + 1)**2)
plt.plot(t, parabolic_mask)
plt.plot(t, parabolic)
```

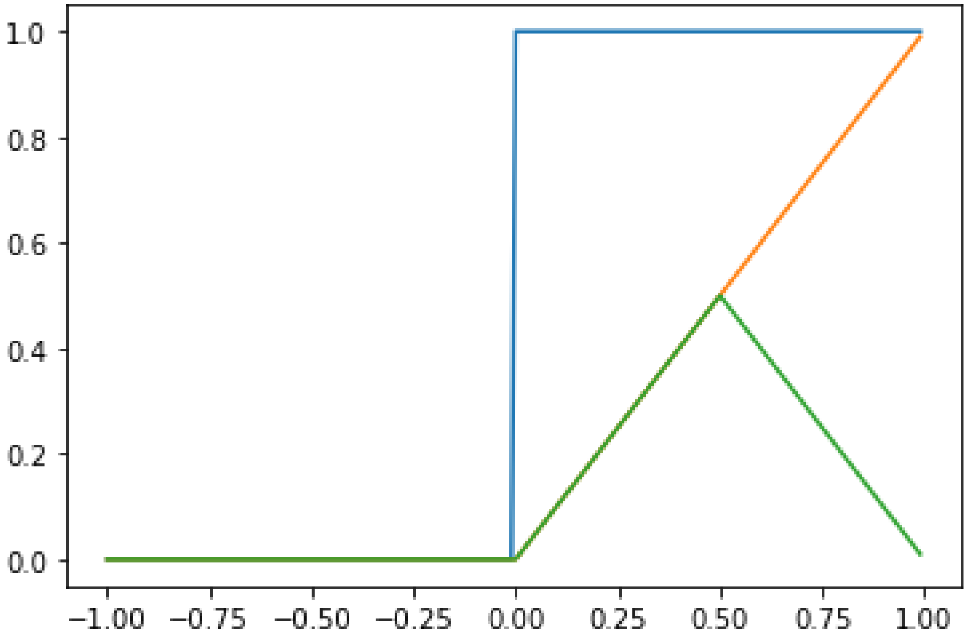
[12]:

```
[<matplotlib.lines.Line2D at 0x7f7105b032b0>]
```



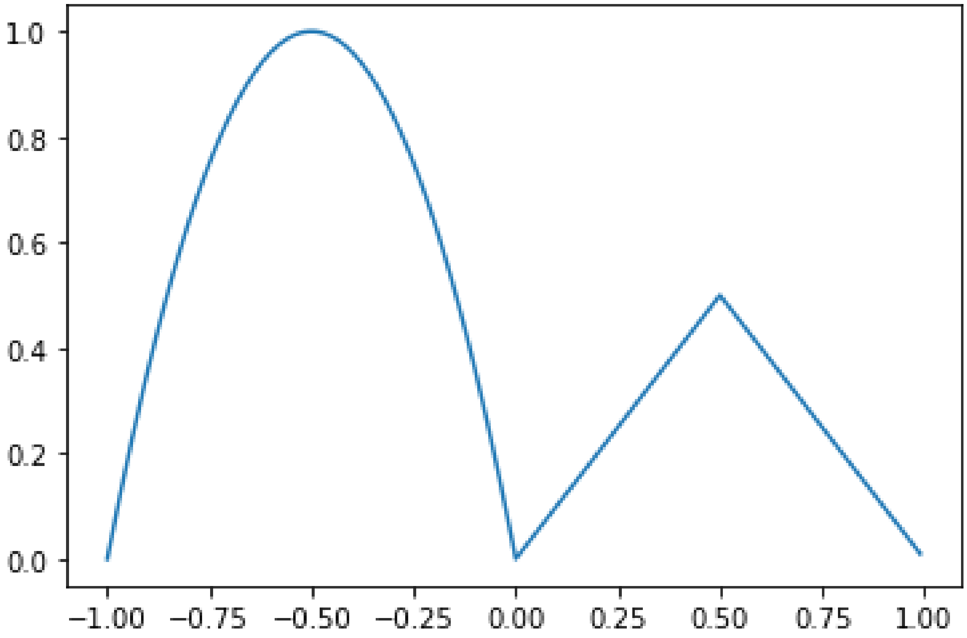
[19]:
```
triangle_mask = t > 0
triangle = triangle_mask * t
plt.plot(t, triangle_mask)
plt.plot(t, triangle)
triangle[triangle > 0.5] = -triangle[triangle > 0.5] + 1
plt.plot(t, triangle)
```

[19]:
```
[<matplotlib.lines.Line2D at 0x7f7105911a60>]
```



[20]:
```
y = parabolic + triangle
plt.plot(t, y)
```

[20]:
```
[<matplotlib.lines.Line2D at 0x7f7105866f40>]
```



```
[95]: n = np.arange(1, 25)
      omega = 2 * np.pi * n / T
```

```
[96]: y = y.reshape(1, -1)
      t = t.reshape(1, -1)
      omega = omega.reshape(-1, 1)
```

```
[97]: a = np.trapz(y * np.cos(omega * t), dx=dt, axis=1) * 2 / T
```

```
[98]: b = np.trapz(y * np.sin(omega * t), dx=dt, axis=1) * 2 / T
```

```
[99]: a0 = np.trapz(y, dx=dt) * 2 / T
```

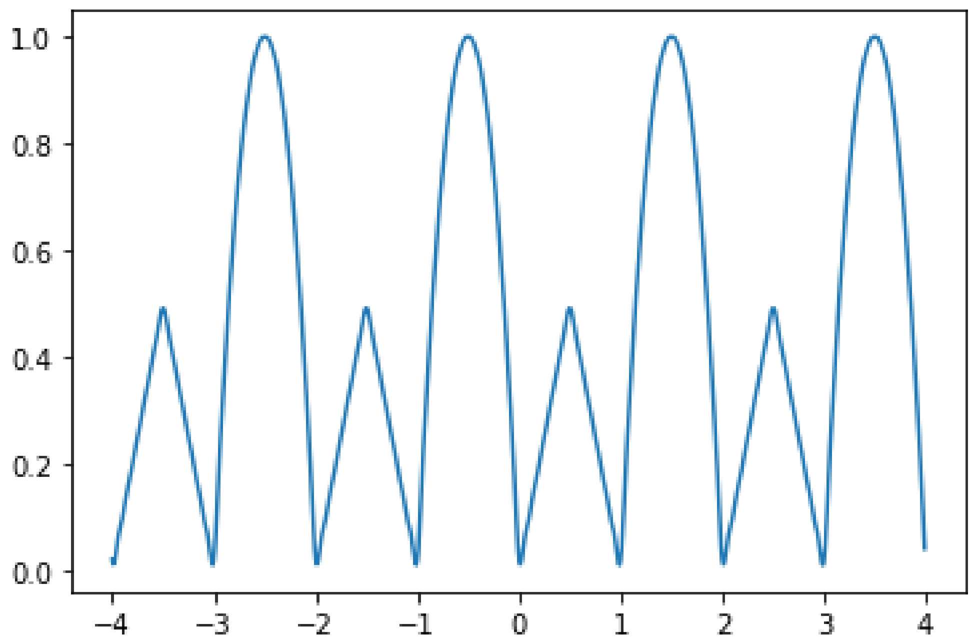
```
[100]: t_fs = np.arange(-2 * T, 2 * T, dt)
```

```
[101]: a = a.reshape(-1, 1)
       b = b.reshape(-1, 1)
```

```
[102]: y_fs = a0 / 2 + np.sum(a * np.cos(omega * t_fs) + b * np.sin(omega * t_fs),␣
        ↪axis=0)
```

```
[103]: plt.plot(t_fs, y_fs)
```

```
[103]: [<matplotlib.lines.Line2D at 0x7f71053f2b80>]
```



```
[110]: omega0 = 5
```

```
[111]: lowpass_mag = omega0 / np.sqrt(omega0**2 + omega**2)
       lowpass_mag
```

```
[111]: array([[0.84673302],
              [0.62267699],
              [0.46864979],
              [0.36969785],
              [0.30331447],
              [0.25639146],
              [0.22170592],
              [0.19511986],
              [0.17413698],
              [0.15717673],
              [0.14319524],
              [0.13147778],
              [0.12151958],
              [0.11295455],
              [0.10551104],
              [0.09898334],
              [0.09321295],
              [0.0880758 ],
              [0.08347342],
              [0.0793267 ],
              [0.07557134],
              [0.07215459],
              [0.06903272],
              [0.06616923]])
```

```
[112]: lowpass_phase = np.arctan(omega / omega0)
       lowpass_phase
```

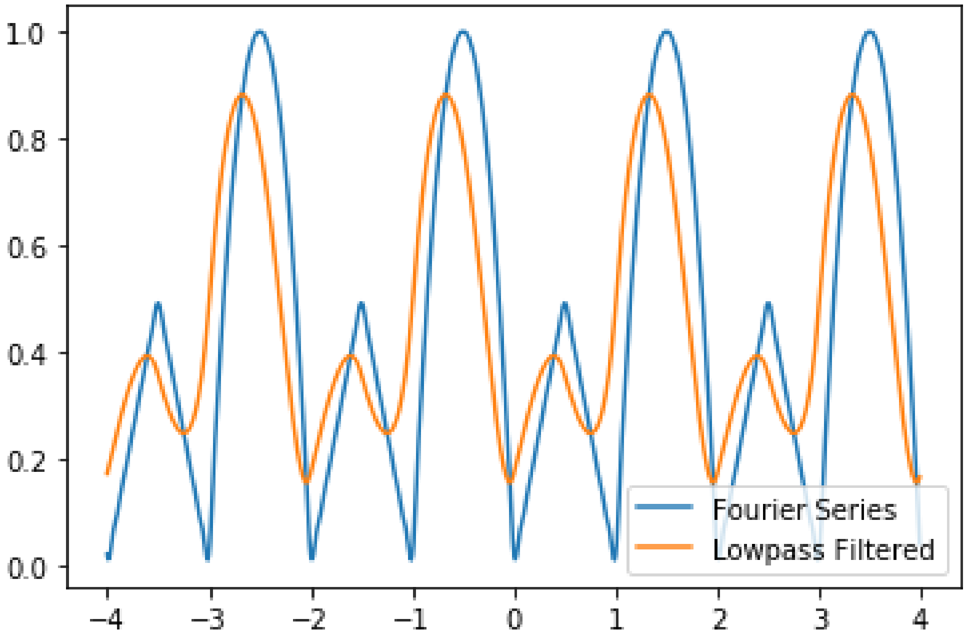
```
[112]: array([[0.56098212],
              [0.89863709],
              [1.08303462],
              [1.19211252],
              [1.26262726],
              [1.31150932],
              [1.34723274],
              [1.37441667],
              [1.39576703],
              [1.41296514],
              [1.42710715],
              [1.43893677],
              [1.44897566],
              [1.45760019],
              [1.46508853],
              [1.47165063],
              [1.47744786],
              [1.48260626],
              [1.48722567],
              [1.4913862 ],
              [1.49515287],
              [1.49857898],
              [1.50170866],
              [1.50457872]])
```

```
[113]: a_lowpass = a * lowpass_mag
       b_lowpass = b * lowpass_mag
```

```
[114]: y_lowpass = a0 / 2 + np.sum(a_lowpass * np.cos(omega * t_fs + lowpass_phase) +␣
         ↪b_lowpass * np.sin(omega * t_fs + lowpass_phase), axis=0)
```

```
[115]: plt.plot(t_fs, y_fs, label="Fourier Series")
       plt.plot(t_fs, y_lowpass, label="Lowpass Filtered")
       plt.legend()
```

```
[115]: <matplotlib.legend.Legend at 0x7f7105332c40>
```


1.0
0.8
0.6
0.4
0.2
0.0
−4
−3
−2
−1
0
1
2
3
4
Fourier Series
Lowpass Filtered

[ ]:

[ ]: