

11.9.13

Obtain $F(e^{-x^2/2})$ using table

$$F(e^{-ax^2}) = \frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}} \quad a > 0$$

$$e^{-x^2/2} = e^{-ax^2} \Rightarrow a = \frac{1}{2}$$

$$F(e^{-x^2/2}) = \frac{1}{\sqrt{2 \cdot \frac{1}{2}}} e^{-\frac{\omega^2}{4 \cdot \frac{1}{2}}} = e^{-\frac{\omega^2}{2}}$$

$$\begin{aligned} \text{find } \frac{d}{dx} e^{-x^2/2} &= F^{-1}(i\omega F(e^{-x^2/2})) \\ &= F^{-1}(i\omega e^{-\omega^2/2}) \end{aligned}$$

$$\frac{d}{dx} e^{-x^2/2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} i\omega e^{-\omega^2/2} e^{i\omega x} d\omega$$

$$\equiv \int_{-\infty}^{\infty} i\omega e^{-\omega^2/2} e^{i\omega x} d\omega = i\omega e^{-\omega^2/2} - \int_{-\infty}^{\infty} \omega^2 e^{-\omega^2/2} d\omega$$