

11, 9, 16

part a.

$$F(f(x+a)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x+a) e^{-i\omega x} dx$$

$$\text{let } v = x+a$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(v) e^{-i\omega(v-a)} dv$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(v) e^{-i\omega v} e^{i\omega a} dv$$

$$= e^{i\omega a} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(v) e^{-i\omega v} dv$$

$$= e^{i\omega a} F(f(x))$$

part c

$$F^{-1}(F(\omega+a)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega+a) e^{i\omega x} d\omega$$

$$v = \omega+a$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(v) e^{i(v-a)x} dv$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(v) e^{ivx} e^{-iax} dv$$

$$= e^{-iax} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(v) e^{ivx} dv = e^{-iax} F^{-1}(F(\omega))$$