Partial Differential equ's

Partial Derivatives

normaly $\frac{d}{dt} f(t)$ for example $\frac{d}{dt} at^2+bt+c$

what if $\frac{d}{dt} f(t, x)$ $\frac{d}{dt} axt^2$

need to find $x = g(t)$

partial derivative $\frac{\partial}{\partial t} f(t, x) = \partial_t f(t, x)$

$$\frac{\partial}{\partial t} axt^2 = 2axt$$

PDE $\frac{\partial u}{\partial t} = \alpha \nabla^2 u$

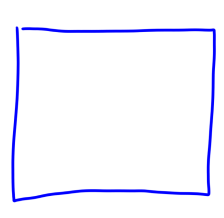
if $u(\mathbb{R}, \mathbb{R}) \to \mathbb{R}$ $u(t, x)$

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

if $u(\mathbb{R}, \mathbb{R}^3) \to \mathbb{R}$

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

in 2D case



what is the heat at every point

PDE vs ODE

PDE has derivatives WRT multiple variables

ODE have derivatives WRT a single variable

$$\frac{d^2 i}{dt^2} + a_1 \frac{di}{dt} + a_0 = i_s(t) \quad \text{ODE}$$

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} \quad \text{PDE}$$

at steady state $\frac{\partial u}{\partial t} = 0$

$$\frac{\partial^2 u}{\partial x^2} = 0 \quad \text{ODE}$$

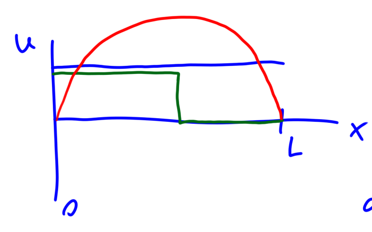
$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

at steady state

$$0 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \quad \text{PDE}$$

Boundary Conditions and Initial conditions

Intial Conditions



heat eq'n in 1D.

at $t=0$

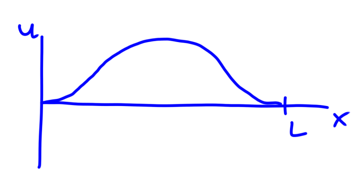
$u(t, x)$

IC: $u(0, x) = 5$

$u(0, x) = \begin{cases} 4 & x < \frac{L}{2} \\ 0 & x \geq \frac{L}{2} \end{cases}$

$u(0, x) = 8 - (x - \frac{L}{2})^2$

Boundary Conditions

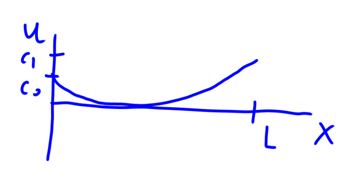


no flow

$\left. \frac{\partial u}{\partial x} \right|_{x=0,L} = 0$

perfectly insulated

Dirischlet

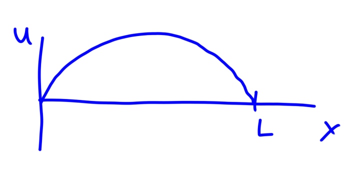


constant heat

$u(t, 0) = c_0$

$u(t, L) = c_1$

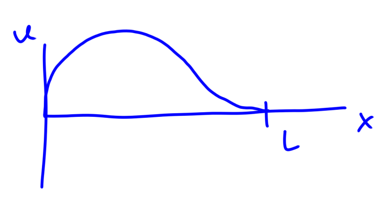
robin



$u(t, 0) = 0$

$u(t, L) = 0$

neumann



neumann on left

$u(t, 0) = 0$

dirichlet

$\left. \frac{\partial u}{\partial x} \right|_{x=L} = 0$

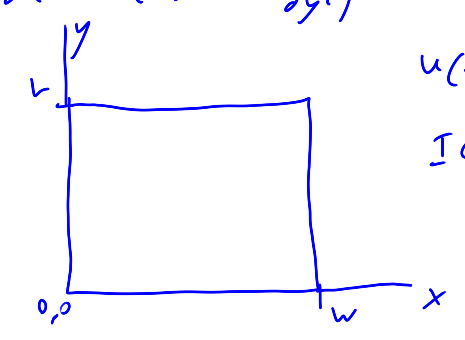
time varying boundary conditions

$u(t, 0) = \cos(t)$

$\left. \frac{\partial u}{\partial x} \right|_{x=0} + u(t, 0) = 0$

Boundary conditions in 2D

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$



$u(t, x, y)$

IC: $u(0, x, y)$

what is the heat a every point at $t=0$

Boundary conditions

$u(t, x, 0)$ bottom edge

$u(t, x, L)$ top edge

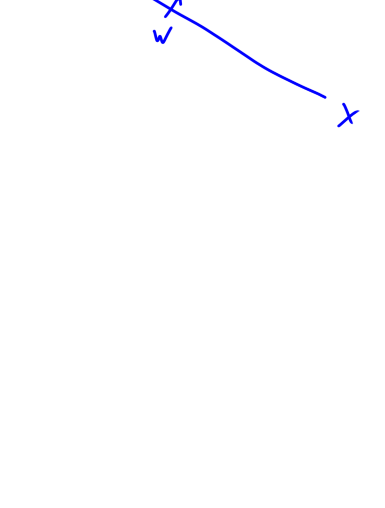
$u(t, 0, y)$ left edge

$u(t, W, y)$ right edge

For example $u(t, x, 0) = \frac{x}{W}$

linearly varies from 0 to 1 from $x=0$ to $x=W$

3D boundary condition



$u(t, x, y, z)$

top face

$u(t, x, y, H) = f(t, x, y)$

etc

$u(t, x, 0, z)$

x-z plane at $y=0$

6 boundary conditions