11-17_PDE_solution_animations

September 27, 2023

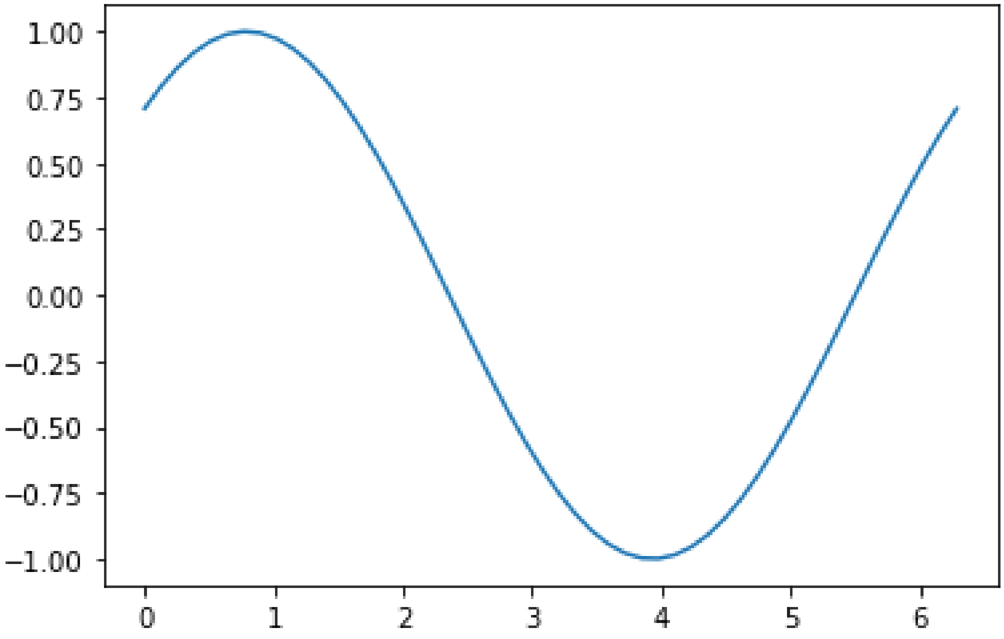
[2]:
```
import matplotlib.pyplot as plt
import numpy as np
%matplotlib inline
```

[3]:
```
from matplotlib.animation import FuncAnimation
from IPython import display
```

[6]:
```
X = np.linspace(0, np.pi * 2)
phi = np.pi / 4
Y = np.sin(X + phi)
```

[7]:
```
plt.plot(X, Y)
```

[7]:
```
[<matplotlib.lines.Line2D at 0x7efc71e8e1f0>]
```



[13]:

```
fig, ax = plt.subplots(figsize=(7,3))

line, = ax.plot([])

ax.set_xlim(0, 2 * np.pi)
ax.set_ylim(-1.1, 1.1)

anim = FuncAnimation(
  fig,
  lambda frame: line.set_data((X, np.sin(X + 2 * np.pi * frame / 100))),
  frames=100,
  interval=50
)

video = anim.to_html5_video()
html = display.HTML(video)
display.display(html)
plt.close()
```

```
<IPython.core.display.HTML object>
```

[19]:

```
def animate(x, fx, xlim=None, ylim=None, frames=100, interval=20, max_t=1):
  fig, ax = plt.subplots(figsize=(7,3))

  line, = ax.plot([])

  if xlim is not None:
    ax.set_xlim(xlim)
  if ylim is not None:
    ax.set_ylim(ylim)

  anim = FuncAnimation(
    fig,
    lambda frame: line.set_data((x, fx(max_t * frame / frames))),
    frames=frames,
    interval=interval
  )

  video = anim.to_html5_video()
  html = display.HTML(video)
  display.display(html)
  plt.close()
```

[20]:

```
animate(
  X,
  lambda t: np.sin(X + t),
  xlim=(0, 2 * np.pi),
  ylim=(-1.1, 1.1),
  max_t=2 * np.pi,
)
```

```
<IPython.core.display.HTML object>
```

[62]:

```
L = 1
k = 1
N = 100
x = np.linspace(0, L, 300)
x = x.reshape(1, -1)
n = np.arange(1, N, 2)
n = n.reshape(-1, 1)

def heat(t):
  return np.sum(4 / (n * np.pi) * np.exp(-k * (n * np.pi / L)**2 * t) * np.
 ↪sin(n * np.pi * x / L), axis=0)
```

[63]:

```
heat(0).shape
```

[63]:

```
(300,)
```

[39]:

```
animate(
  x,
  heat,
  xlim=(0, L),
  ylim=(0, 1.1),
  max_t=0.2,
)
```

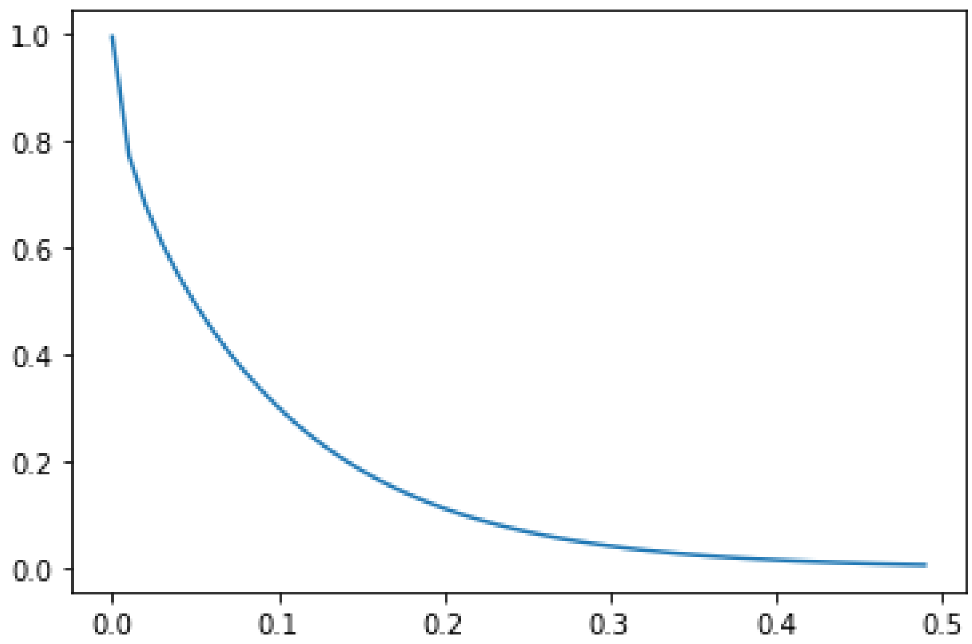
```
<IPython.core.display.HTML object>
```

[65]:

```
t = np.arange(0, 0.5, 0.01)
avg = np.zeros_like(t)
for i, ti in enumerate(t):
  avg[i] = np.trapz(heat(ti), dx=np.diff(x)[0]) / L

plt.plot(t, avg)
```

[65]:

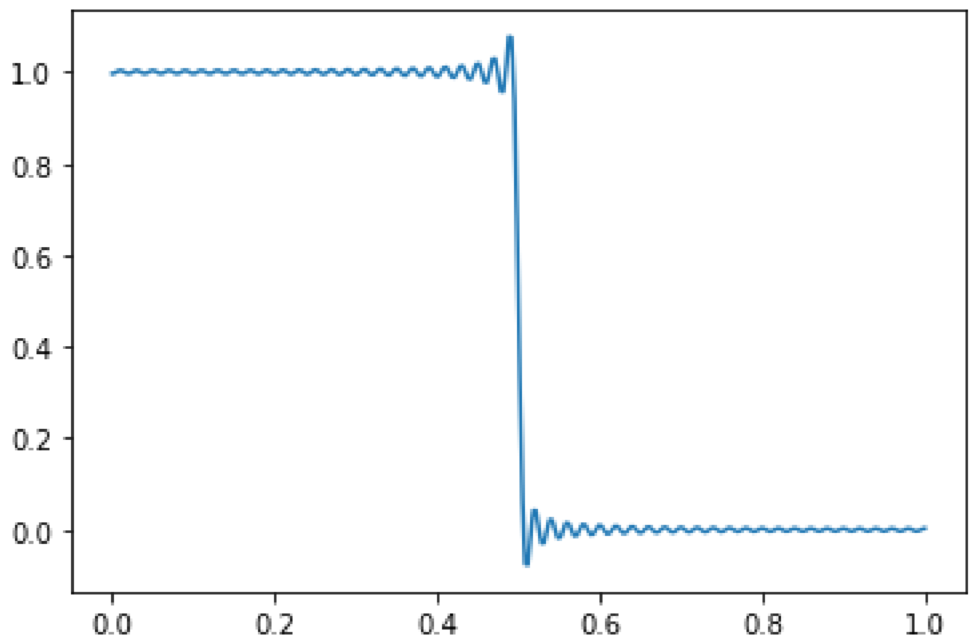
```
[<matplotlib.lines.Line2D at 0x7efc701aa820>]
```



[51]:
```
n = np.arange(1, N)
n = n.reshape(-1, 1)

a0 = 1

def heat(t):
  return a0 / 2 + np.sum(2 / (n * np.pi) * np.sin(n * np.pi / 2) * np.exp(-k *␣
 ↪(n * np.pi / L)**2 * t) * np.cos(n * np.pi / L * x), axis=0)
```

[52]:
```
plt.plot(x.flatten(), heat(0))
```

[52]:
```
[<matplotlib.lines.Line2D at 0x7efc7019f6d0>]
```

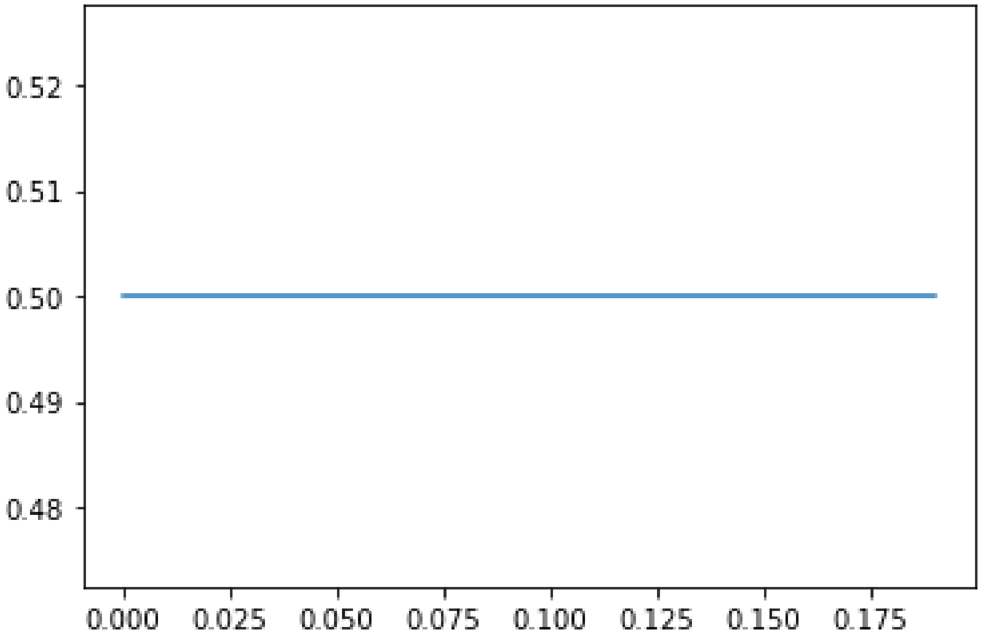


[58]:
```
animate(
  x,
  heat,
  xlim=(0, L),
  max_t=0.5,
  frames=200
)
```

```
<IPython.core.display.HTML object>
```

[60]:
```
t = np.arange(0, 0.2, 0.01)
avg = np.zeros_like(t)
for i, ti in enumerate(t):
  avg[i] = np.trapz(heat(ti), dx=np.diff(x)[0]) / L

plt.plot(t, avg)
```

[60]:
```
[<matplotlib.lines.Line2D at 0x7efc67f865e0>]
```



[92]:

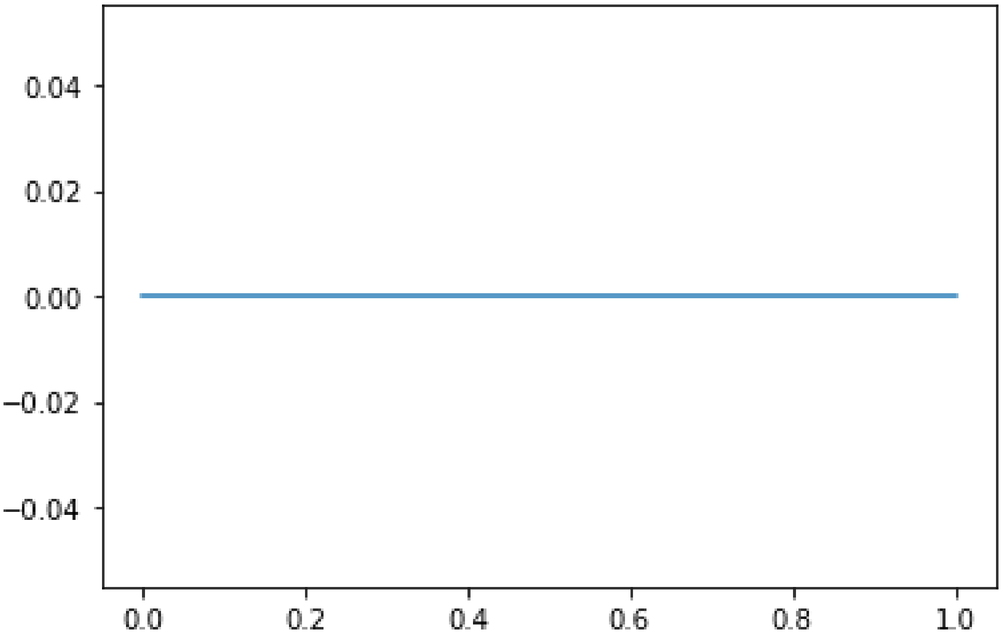
```
Delta = 0.1
L = 1
c = 1
N = 150

x = np.linspace(0, L, 150)
x = x.reshape(1, -1)

n = np.arange(1, N)
n = n.reshape(-1, 1)

def u(t):
  return np.sum(2 / (c * n * np.pi) * np.sin(n * np.pi * Delta) * np.sin(n * np.
 ↪pi / L * x) * np.sin(c * n * np.pi / L * t), axis=0)
```

[93]:

```
plt.plot(x.flatten(), u(0))
```

[93]:

```
[<matplotlib.lines.Line2D at 0x7efc70a808b0>]
```



[94]:

```
animate(
  x,
  u,
  max_t=2,
  ylim=(-0.6, 0.6),
  frames=500,
)
```

```
<IPython.core.display.HTML object>
```

[95]:

```
def u(t):
  return np.sum(2 / (c * n * np.pi) * np.cos(n * np.pi * Delta) * np.sin(c * n␣
↪* np.pi / L * t) * np.cos(n * np.pi / L * x), axis=0)
```

[96]:

```
Delta = 0

animate(
  x,
  u,
  max_t=2,
  ylim=(-1.1, 1.1),
  frames=400,
)
```

```
<IPython.core.display.HTML object>
```

[ ]:

```

```