11-19_PDE_Simulation

November 19, 2021

```
[1]: import numpy as np
     import matplotlib.pyplot as plt
     from scipy.integrate import odeint
     %matplotlib inline
```

```
[2]: L = 1
     alpha = 0.5
     dt = 0.001
     dx = 0.01
     x = np.arange(0, L, dx)
     t = np.arange(0, 0.5, dt)
```

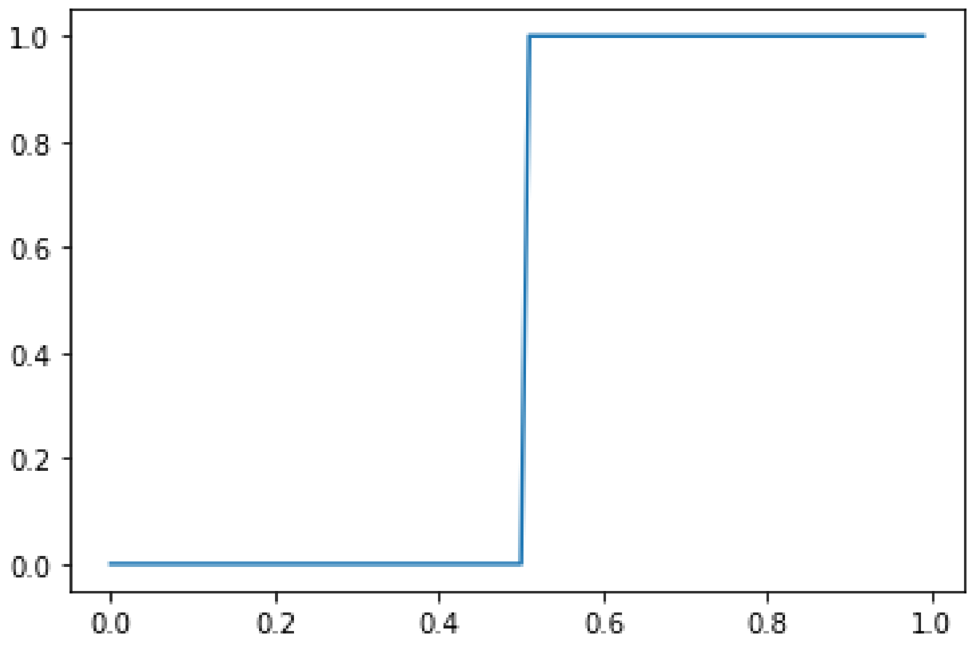
```
[3]: A = -2 * np.diag(np.ones([x.size]))
     A += np.diag(np.ones([x.size - 1]), 1)
     A += np.diag(np.ones([x.size - 1]), -1)
     #A[0,:] = 0
     #A[-1,:] = 0
     A = alpha * A / dx**2
     A
```

```
[3]: array([[-10000.,   5000.,      0., …,      0.,      0.,      0.],
            [  5000., -10000.,   5000., …,      0.,      0.,      0.],
            [     0.,   5000., -10000., …,      0.,      0.,      0.],
            …,
            [     0.,      0.,      0., …, -10000.,   5000.,      0.],
            [     0.,      0.,      0., …,   5000., -10000.,   5000.],
            [     0.,      0.,      0., …,      0.,   5000., -10000.]])
```

```
[4]: def heat(u, t):
       return (A @ u.reshape(-1, 1)).flatten()
```

```
[5]: u0 = np.ones_like(x) * x > 0.5
     plt.plot(x, u0)
```

```
[5]: [<matplotlib.lines.Line2D at 0x7fb4211ee430>]
```



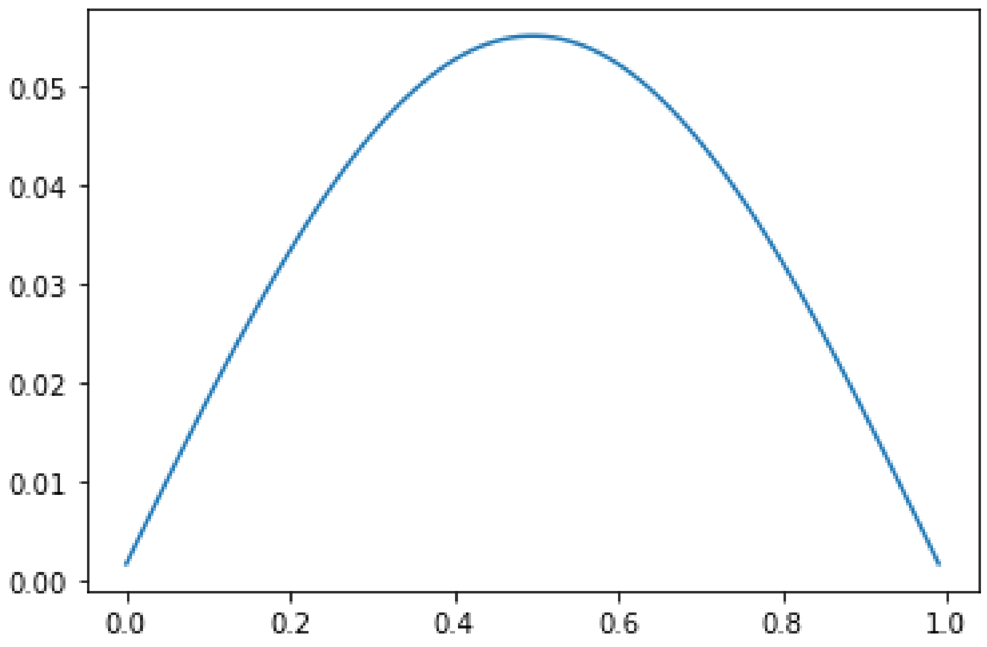
```
[6]: heat(u0, 0)
```

```
[6]: array([    0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,  5000., -5000.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0.,     0.,     0.,     0.,     0.,     0.,
                0.,     0.,     0., -5000.])
```

```
[7]: u = odeint(heat, u0, t)
```

```
[8]: plt.plot(x, u[-1, :])
```

```
[8]: [<matplotlib.lines.Line2D at 0x7fb41f0ff4f0>]
```



[9]:
```python
from matplotlib.animation import FuncAnimation
from IPython import display

def animate(x, u, interval=20, ylim=None):
  fig, ax = plt.subplots(figsize=(7,3))

  line, = ax.plot([])

  ax.set_xlim(x.min(), x.max())
  if ylim is None:
    ax.set_ylim(u.min(), u.max())
  else:
    ax.set_ylim(ylim)

  anim = FuncAnimation(
    fig,
    lambda frame: line.set_data((x, u[frame, :])),
    frames=u.shape[0],
    interval=interval
  )

  video = anim.to_html5_video()
  html = display.HTML(video)
  display.display(html)
  plt.close()
```

[10]:
```
animate(x, u, 20)
```

```
<IPython.core.display.HTML object>
```

[11]:
```
An = np.array(A)
An[0,:] = An[1,:]
An[-1,:] = An[-2,:]
An
```

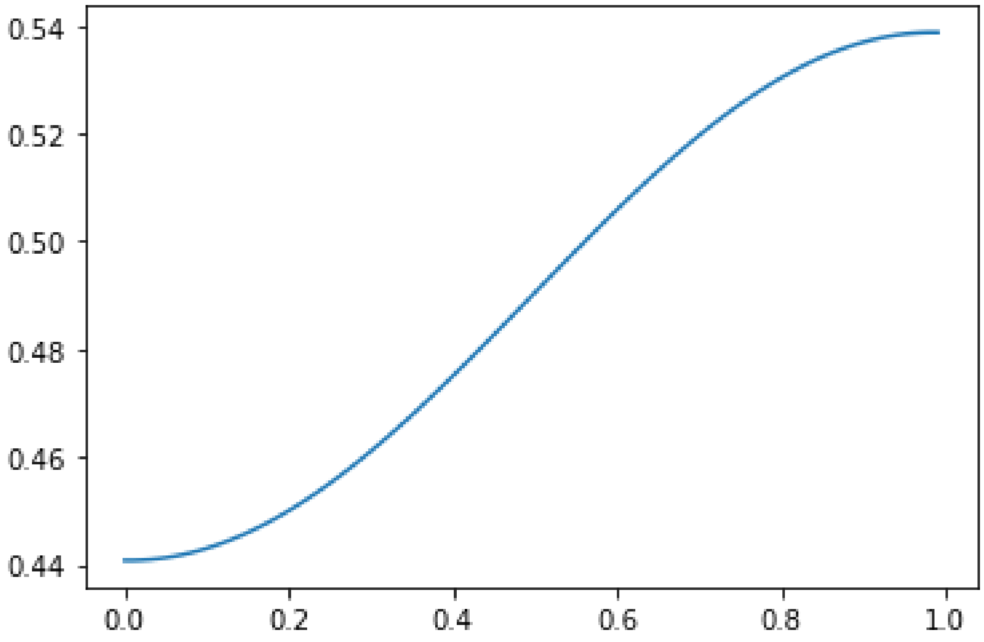
[11]:
```
array([[  5000., -10000.,   5000., ...,      0.,      0.,      0.],
       [  5000., -10000.,   5000., ...,      0.,      0.,      0.],
       [     0.,   5000., -10000., ...,      0.,      0.,      0.],
       ...,
       [     0.,      0.,      0., ..., -10000.,   5000.,      0.],
       [     0.,      0.,      0., ...,   5000., -10000.,   5000.],
       [     0.,      0.,      0., ...,   5000., -10000.,   5000.]])
```

[12]:
```
def heat_neumann(u, t):
  u[0] = u[1]
  u[-1] = u[-2]
  return (An @ u.reshape(-1, 1)).flatten()
```

[13]:
```
u = odeint(heat_neumann, u0, t)
```

[14]:
```
plt.plot(x, u[-1,:])
```

[14]:
```
[<matplotlib.lines.Line2D at 0x7fb43e2a5040>]
```

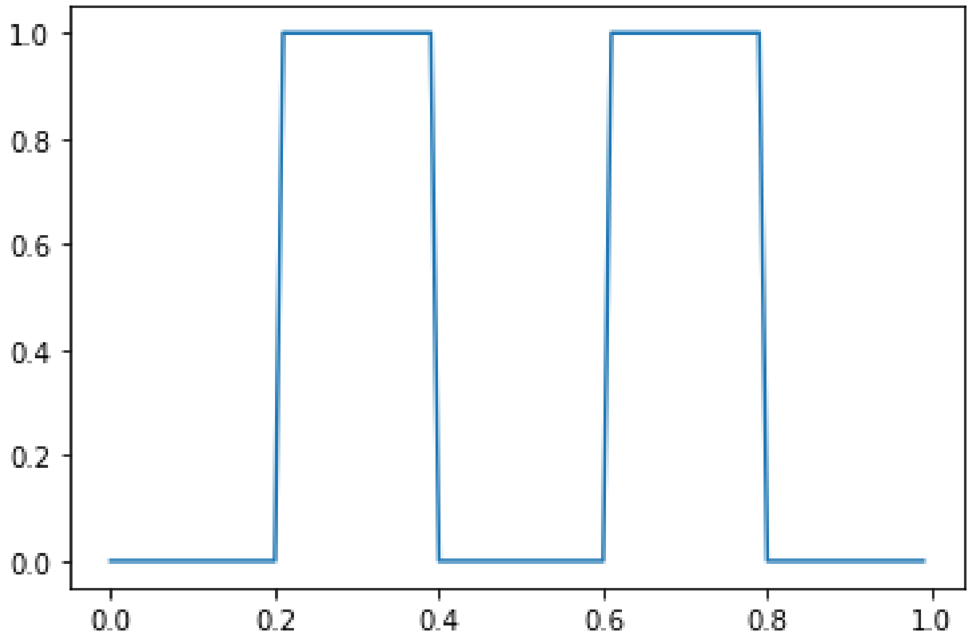


[15]:
```
animate(x, u,)
```

```
<IPython.core.display.HTML object>
```

[16]:
```
u0_2 = np.ones_like(x) * ((x * 5 * L) % 2 > 1)
plt.plot(x, u0_2)
```

[16]:
```
[<matplotlib.lines.Line2D at 0x7fb43e1d1160>]
```



[17]:
```
u = odeint(heat_neumann, u0_2, t)
animate(x, u)
```

```
<IPython.core.display.HTML object>
```

[18]:
```
u = odeint(heat, u0_2, t)
animate(x, u)
```

```
<IPython.core.display.HTML object>
```

[19]:
```
def heat_t(u, t):
  u[0] = u[1]
  u[-1] = np.sin(8 * np.pi * t)
  return (An @ u.reshape(-1, 1)).flatten()
```

[20]:
```
u0_t = np.zeros_like(x)
```

[21]:
```
u = odeint(heat_t, u0_t, t)
```

[22]:
```
animate(x, u)
```

```
<IPython.core.display.HTML object>
```

[ ]:
```
```