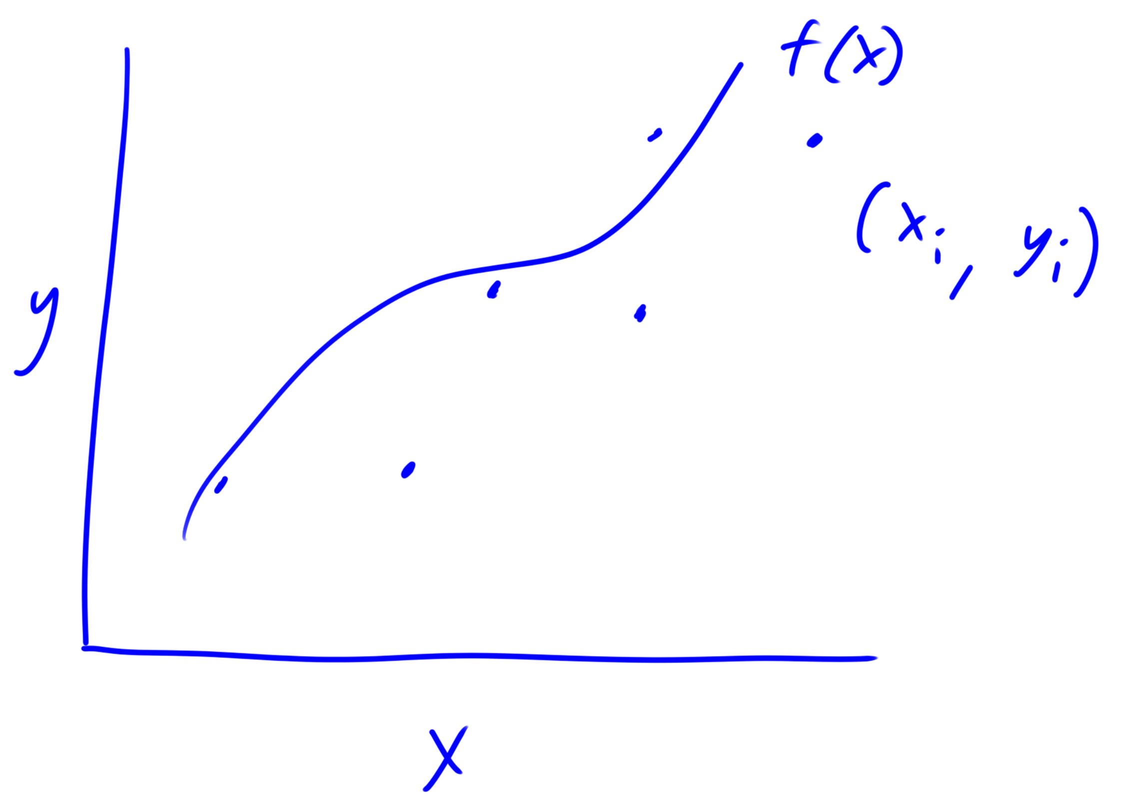

n points

$$f(x) = a_1 f_1(x) + a_2 f_2(x) + \dots + a_m f_m(x)$$

$$y_i = a_1 f_1(x_i) + \dots a_m f_m(x_i)$$

$$\varepsilon_i = a_1 f_1(x_i) + \dots + a_m f_m(x_i) - y_i$$

$$\vec{x} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \qquad \vec{x} = \underset{\vec{x}}{\operatorname{argmin}} \left\| \begin{matrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{matrix} \right\|_2^2$$

$$A = \begin{bmatrix} f_1(x_1) & f_2(x_1) & \cdots & f_m(x_1) \\ f_1(x_2) & f_2(x_2) & \cdots & f_m(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ f_1(x_n) & f_2(x_n) & \cdots & f_m(x_n) \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \qquad \vec{x} = \underset{\vec{x}}{\operatorname{argmin}} \| A\vec{x} - y \|_2^2$$

$$A\vec{x} = y$$

$$\vec{x} = A^{-1} y \quad \times \qquad n \neq m$$

Moore Penrose Pseudo Inverse

normal inverse $A^{-1}A = I$

$$AA^+A = A$$

$$A^+AA^+ = A^+$$

$$(AA^+)^T = AA^+$$

$$(A^+A)^T = A^+A$$

$$A^+ = (A^TA)^{-1}A^T$$

$$\boxed{\vec{x} = (A^TA)^{-1}A^Ty}$$

in Matlab $\vec{x} = A \backslash y$

linear example

$$y = mx + b$$

$f_1(x) = x \qquad a_1 = m$

$f_2(x) = 1 \qquad a_2 = b$

$$A = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}$$

RC circuit

$V_0 = 1 - e^{-t/\tau}$ find τ

$$e^{-t/\tau} = 1 - V_0$$

$$-\frac{t}{\tau} = \ln(1 - V_0)$$

$$\frac{t}{\tau} = -\ln(1 - V_0) \qquad y = -\ln(1 - V_0)$$

$$y = \begin{bmatrix} -\ln(1 - V_{0_1}) \\ -\ln(1 - V_{0_2}) \\ \vdots \\ -\ln(1 - V_{0_n}) \end{bmatrix} \qquad A = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix} \qquad \vec{x} = [1/\tau]$$

Sinusoids

$$y = a_1 \cos(4\pi t) + a_2 \sin(6\pi t)$$

$$A = \begin{bmatrix} \cos(4\pi t_1) & \sin(6\pi t_1) \\ \cos(4\pi t_2) & \sin(6\pi t_2) \\ \vdots & \vdots \\ \cos(4\pi t_n) & \sin(6\pi t_n) \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$