Eigen Decomposition

$A \in \mathbb{R}^{n\times n}$

$A = Q\Lambda Q^{-1}$ $\quad Q = \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{bmatrix}$

$Av_i = \lambda_i v_i$ $\quad \Lambda = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$ $\quad \lambda_i \in \mathbb{C}$

if $A \in \mathbb{R}^{n\times m}$

Singular Value Decomposition (SVD)

$A = U\Sigma V^*$ $\quad$ * complec conjugate transpose

transpose $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

complex conjugate $\quad a+bi \to a-bi$ $\quad a,b \in \mathbb{R}$

$\Sigma \in \mathbb{R}^{n\times m}$ $\quad U \in \mathbb{C}^{n\times n}$ $\quad V \in \mathbb{C}^{m\times m}$

$$\begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} & & \\ & U & \\ & & \end{bmatrix}\begin{bmatrix} & & \\ & \Sigma & \\ & & \end{bmatrix}\begin{bmatrix} V \end{bmatrix}$$

$A = \begin{bmatrix} \hat{U} & \hat{U}^{\perp} \end{bmatrix}\begin{bmatrix} \hat{\Sigma} \\ 0 \end{bmatrix} V^* = \hat{U}\hat{\Sigma}V^*$ $\quad$ since Σ diag

the values in Σ are sorted

$A^{-1} = V\hat{\Sigma}^{-1}\hat{U}^*$ $\quad Ax = b$ $\quad x = V\hat{\Sigma}^{-1}\hat{U}^*b$

$\hat{\Sigma} = \begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \ddots & \\ 0 & & & \sigma_n \end{bmatrix}$ $\quad \hat{\Sigma}^{-1} = \begin{bmatrix} 1/\sigma_1 & & & 0 \\ & 1/\sigma_2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_n \end{bmatrix}$

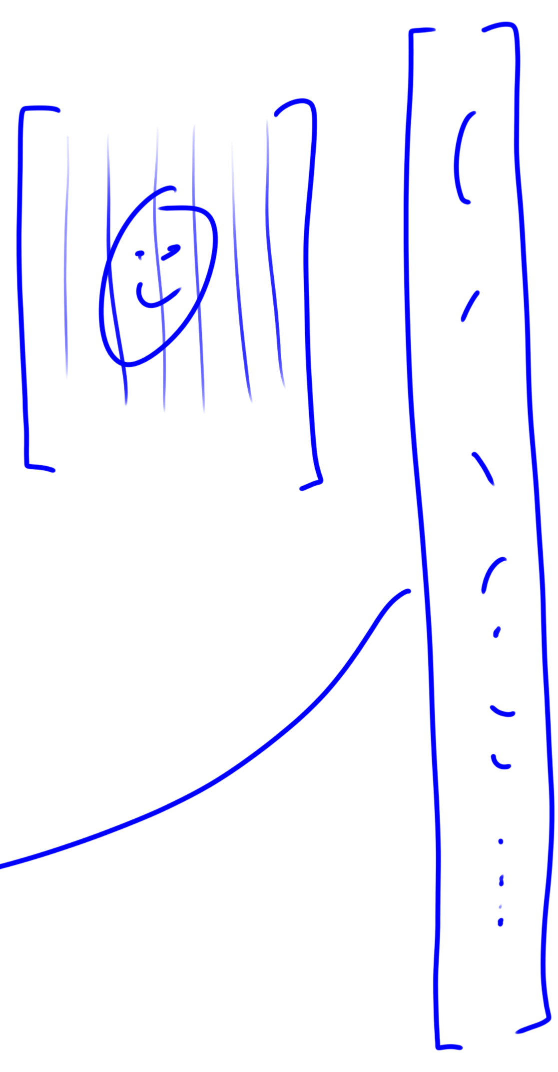
$$A = \begin{bmatrix} \boxed{\hat{U}} & & \end{bmatrix}\begin{bmatrix} \boxed{} & \\ \hat{\Sigma} & \end{bmatrix}\begin{bmatrix} \boxed{V^*} \\ \end{bmatrix}$$

$U = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_n \\ | & | & & | \end{bmatrix}$

Data Driven Science and Engineering

Principal Component Analysis (PCA)

$\mathbb{X} = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \cdots & x_n \\ | & | & & | \end{bmatrix}$



$\mathbb{X} \in \mathbb{R}^{m\times n}$

$SVD(\mathbb{X} - mean(\mathbb{X}))$

$mean(\mathbb{X})_j = \frac{1}{n}\sum_{i=0}^{n} x_{ij} = \begin{bmatrix} | \\ | \\ | \end{bmatrix}$

$\mathbb{X} = \begin{bmatrix} | & | & & | \\ u_1 & u_2 & \cdots & u_n \\ | & | & & | \end{bmatrix}\begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_n \end{bmatrix}\begin{bmatrix} — & v_1 & — \\ — & v_2 & — \\ & \vdots & \\ — & v_n & — \end{bmatrix}$

$\mathbb{X} = \hat{U}\hat{\Sigma}V^*$

images

$\hat{U}^*\mathbb{X} = \hat{U}^*\hat{U}\hat{\Sigma}V^*$

$\hat{U}^*\mathbb{X} = \hat{\Sigma}V^*$

$\hat{\Sigma}^{-1}\hat{U}^*\mathbb{X} = \hat{\Sigma}^{-1}\hat{\Sigma}V^*$

$= V^*$

$= \begin{bmatrix} — & v_1 & — \\ — & v_2 & — \end{bmatrix}$