nlin.ss Nonlinear state–space models

1 A state-space model has the general form

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \quad (1a)$$

$$\mathbf{y} = g(x, u, t) \quad (1b)$$

$$\frac{dx}{dt} = Ax + Bu$$

where $\mathbf{f}$ and $\mathbf{g}$ are vector-valued functions that depend on the system. Nonlinear state-space models are those for which $\mathbf{f}$ is a nonlinear functional of either $\mathbf{x}$ or $\mathbf{u}$. For instance, a state variable x_1 might appear as x_1^2 or two state variables might combine as $x_1 x_2$ or an input u_1 might enter the equations as $\log u_1$.

nonlinear state–space models

Autonomous and nonautonomous systems

2 An autonomous system is one for which $\mathbf{f}(\mathbf{x})$, with neither time nor input appearing explicitly. A nonautonomous system is one for which either t or $\mathbf{u}$ do appear explicitly in $\mathbf{f}$. It turns out that we can always write nonautonomous systems as autonomous by substituting in $\mathbf{u}(t)$ and introducing an extra State variable for t[4].

autonomous system

$$\frac{d}{dt}x = f(x)$$

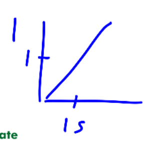
nonautonomous system

$u(t)$ $f(x, u, t)$

3 Therefore, without loss of generality, we will focus on ways of analyzing autonomous systems.

4. Strogatz and Dichter, Nonlinear Dynamics and Chaos.

$$\frac{d}{dt}x = 1$$



Equilibrium

4 An equilibrium state (also called a fixed State) $\bar{\mathbf{x}}$ is one for which $d\mathbf{x}/dt = \mathbf{0}$. In most cases, this occurs only when the input $\mathbf{u}$ is a constant $\bar{\mathbf{u}}$ and, for time-varying systems, at a given time $\bar{t}$. For autonomous systems, equilibrium occurs when the following holds:

equilibrium state

stationary point

$$f(x) = 0 \quad (2)$$

This is a system of nonlinear algebraic equations, which can be challenging to solve for $\bar{\mathbf{x}}$. However, frequently, several solutions—that is, equilibrium states—do exist.