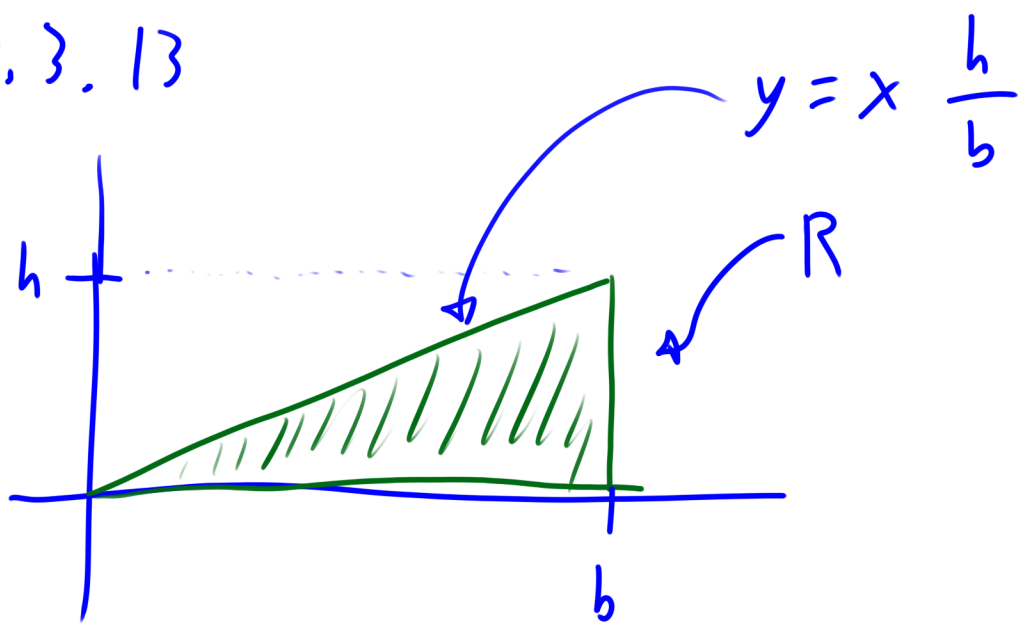


10.3.13



find the center of mass
with density $f(x,y)=1$

$$M = \iint_R f(x,y) dx dy$$

$$= \int_0^b \int_0^{x \frac{h}{b}} 1 dy dx$$

$$= \int_0^b y \Big|_0^{x \frac{h}{b}} dx = \int_0^b x \frac{h}{b} dx$$

$$= \frac{h}{b} \frac{x^2}{2} \Big|_0^b = \frac{h b^2}{2} = \frac{h b}{2} = M$$

$$\bar{x} = \frac{1}{M} \iint_R x f(x,y) dx dy$$

$$= \frac{2}{h b} \int_0^b \int_0^{x \frac{h}{b}} x dy dx$$

$$= \frac{2}{h b} \int_0^b x y \Big|_0^{x \frac{h}{b}} dx = \frac{2}{h b} \int_0^b x^2 \frac{h}{b} dx$$

$$= \frac{2}{b^2} \int_0^b x^2 dx = \frac{2}{b^2} \frac{x^3}{3} \Big|_0^b = \frac{2 b^3}{b^2 3} = \boxed{\frac{2}{3} b = \bar{x}}$$