24.6.1

$f(x) = kx \qquad 0 \le x \le 2$

find mean of $f(x)$
Variance of $f(x)$

$$E(f(x)) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_0^2 x\, f(x)\, dx$$

$$= \int_0^2 kx^2\, dx = k \frac{x^3}{3} \Big|_0^2$$

$$\mu = k \frac{8}{3} = \frac{1}{2} \frac{8}{3} = \boxed{\frac{4}{3} = \mu}$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \int_0^2 (x-\mu)^2 kx\, dx$$

$$= k \int_0^2 x^3 - 2x^2\mu + \mu^2 x\, dx$$

$$= k \left[\frac{x^4}{4} - \frac{2\mu x^3}{3} + \frac{\mu^2 x^2}{2} \right|_0^2$$

$$= k \left(\frac{16}{4} - \frac{2(8)(8)k}{3(3)} + \frac{64k^2 4}{9(2)} \right)$$

$$= \frac{1}{2} \left(4 - \frac{64}{9} + \frac{32}{9} \right)$$

$$= \frac{1}{2} \left(\frac{36}{9} - \frac{64}{9} + \frac{32}{9} \right) = \frac{1}{2} \frac{4}{9} = \boxed{\frac{2}{9} = \sigma^2}$$

$$1 = \int_{-\infty}^{\infty} f(x)\, dx = \int_0^2 kx\, dx = k \frac{x^2}{2} \Big|_0^2$$

$$= k \frac{4}{2} = 2k = 1 \Rightarrow \boxed{k = 1/2}$$