Least Squares

$$f(x) = a_1 f_1(x) + a_2 f_2(x) + \cdots + a_k f_k(x)$$
$$= y$$

n points (x_i, y_i)

$$f(x_i) = y_i = a_1 f_1(x_i) + a_2 f_2(x_i) + \cdots$$

$$\underbrace{\begin{bmatrix} f_1(x_1) & f_2(x_1) & \cdots \\ f_1(x_2) & f_2(x_2) & \cdots \\ \vdots & \vdots & \\ f_1(x_n) & f_2(x_n) & \cdots \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ \vdots \end{bmatrix}}_{\Sigma} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{y}$$

$$A\Sigma = y$$

Moore Penrose Pseudo inverse

$$A^+ = (A^T A)^{-1} A^T \qquad A^+ A = I$$

$$\Sigma = A^+ y = (A^T A)^{-1} A^T y$$

Find m and b in $y = mx + b$ given

x	y
0	1
1	2.1
2	2.9
3	4

$$\Sigma = \begin{bmatrix} m \\ b \end{bmatrix}$$

$$f_1(x) = x$$
$$f_2(x) = 1$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 2.1 \\ 2.9 \\ 4 \end{bmatrix}$$

$$0\,m + b = 1$$