Series

$$\{1, 2, 3\}$$

$$\left\{ \sin(\pi x), \frac{\sin(2\pi x)}{2}, \dots \right\}$$

$$\sum_{n=1}^{\infty} \frac{\sin(n\pi x)}{n} = \sum_{n=1}^{\infty} z_n$$

Check for convergance

$$\lim_{n\to\infty} z_n = c$$

$$\lim_{n\to\infty} \frac{\sin(n\pi x)}{n} = 0$$

Power Series

$$\sum_{n=0}^{\infty} a_n (z-z_0)^n$$

$$\sum_{n=0}^{\infty} \frac{a_n}{n!} (z-z_0)^n$$

Taylor Series

$$f(z) = \sum_{n=1}^{\infty} a_n (z-z_0)^n \qquad a_n = \frac{1}{n!} f^{(n)}(z_0)$$

$$z_0 = 0$$

$$f(z) = \sin(\pi z) \qquad f'(z) = \pi \cos(\pi z)$$

$$a_1 = \frac{1}{1} \pi \cos(\pi z_0) = \pi$$

$$a_2 = \frac{-1}{2} \pi^2 \sin(\pi z_0) = 0$$

$$a_3 = \frac{-1}{3!} \pi^3 \cos(\pi z_0) = \frac{-\pi^3}{3!}$$

$$a_4 = 0$$

$$a_5 = \frac{1}{5!} \pi^5 \cos(\pi z_0) = \frac{\pi^5}{5!}$$

$$a_n = \begin{cases} \frac{\pi^n}{n!} (-1)^{\frac{n-1}{2}} & n \text{ odd} \\ 0 & \text{otherwise} \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} a_n z^n$$

$$f'(z) = \sum_{n=1}^{\infty} n a_n z^{n-1}$$