12-15_Taylor_Series_plots

September 27, 2023

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

[6]:
```
@np.vectorize
def factorial(x):
  if x == 1:
    return x
  else:
    return x * factorial(x - 1)
```

[14]:
```
n = np.arange(1, 6, 2).reshape(-1, 1)
n
```

[14]:
```
array([[1],
       [3],
       [5]])
```

[15]:
```
an = np.pi**n * (-1)**((n-1)/2) / factorial(n)
an
```

[15]:
```
array([[ 3.14159265],
       [-5.16771278],
       [ 2.55016404]])
```

[12]:
```
z = np.arange(-2, 2, 0.01)
```

[19]:
```
fz = np.sum(an * z**n, 0)
fz
```

[19]:
```
array([-4.65467323e+01, -4.51125148e+01, -4.37123212e+01, -4.23455796e+01,
       -4.10117241e+01, -3.97101951e+01, -3.84404386e+01, -3.72019068e+01,
       -3.59940579e+01, -3.48163558e+01, -3.36682704e+01, -3.25492774e+01,
       -3.14588585e+01, -3.03965008e+01, -2.93616975e+01, -2.83539475e+01,
       -2.73727550e+01, -2.64176305e+01, -2.54880895e+01, -2.45836535e+01,
       -2.37038495e+01, -2.28482099e+01, -2.20162728e+01, -2.12075816e+01,
       -2.04216852e+01, -1.96581380e+01, -1.89164997e+01, -1.81963355e+01,
```

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-1.26120376e+01, -1.20838493e+01, -1.15726955e+01, -1.10782034e+01,
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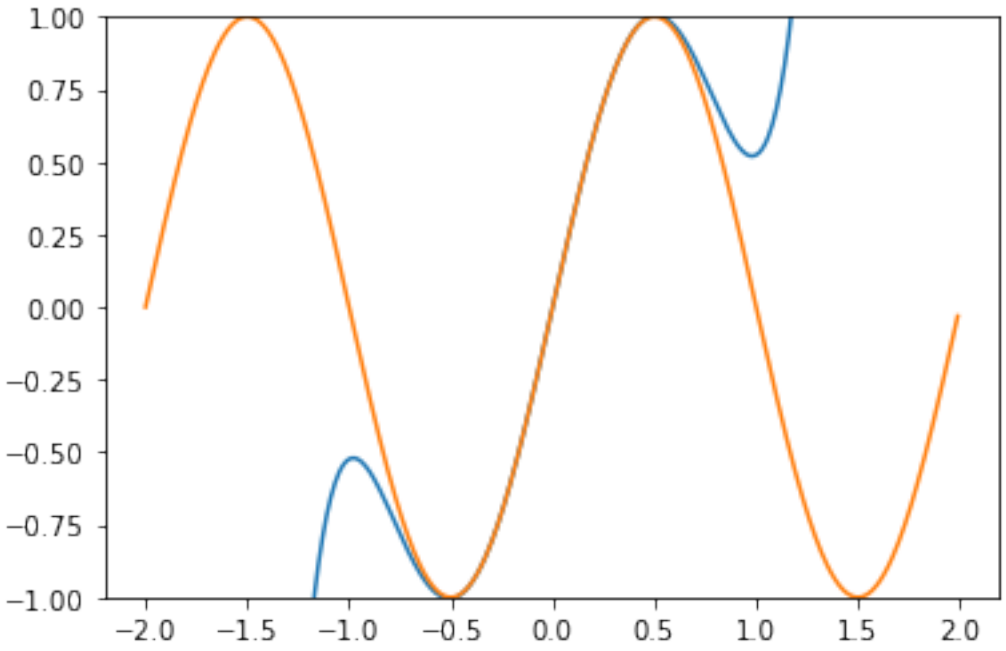
4.10117241e+01,  4.23455796e+01,  4.37123212e+01,  4.51125148e+01])
```

[22]:
```
plt.plot(z, fz)
plt.plot(z, np.sin(np.pi * z))
plt.ylim(-1, 1)
```

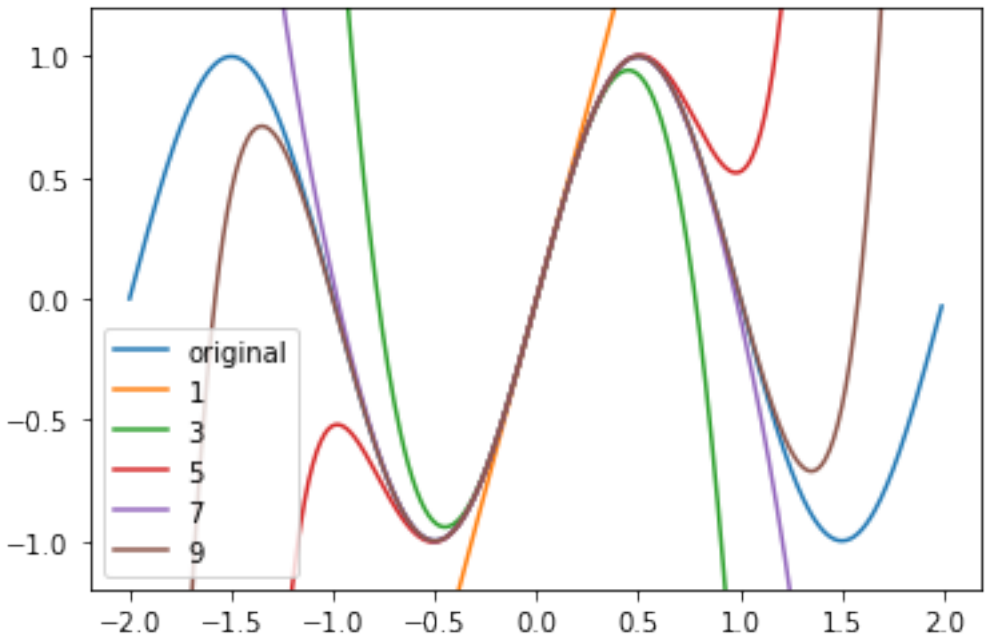
[22]: (-1, 1)



[28]:
```
def estimate(n_max):
  n = np.arange(1, n_max+1, 2).reshape(-1, 1)
  an = np.pi**n * (-1)**((n-1)/2) / factorial(n)
  return np.sum(an * z**n, 0)
```

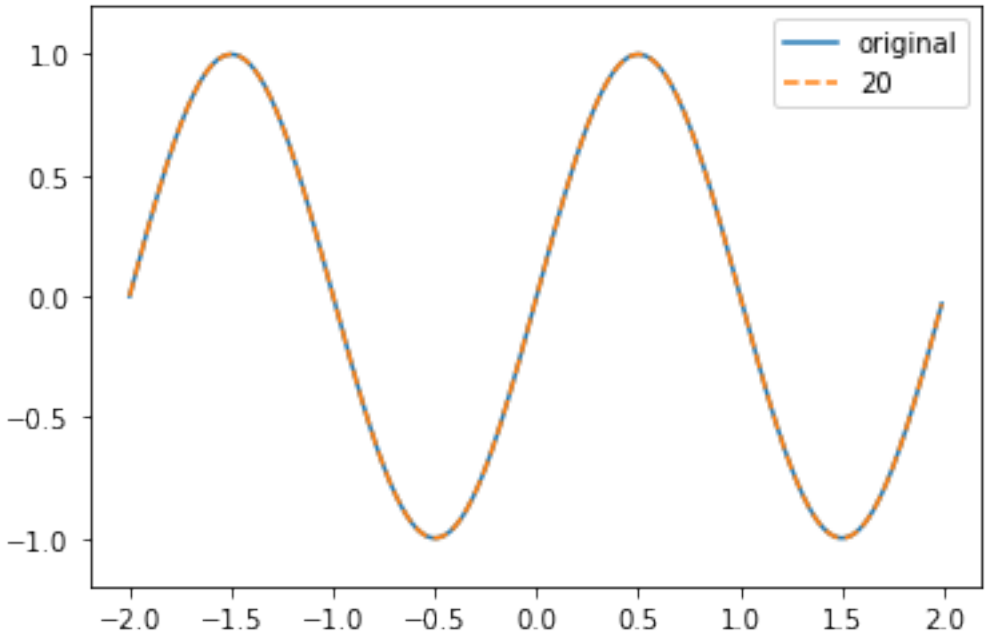
[32]:
```
plt.plot(z, np.sin(np.pi * z), label="original")
for i in range(1, 10, 2):
  plt.plot(z, estimate(i), label=str(i))
plt.ylim(-1.2, 1.2)
plt.legend()
```

[32]: <matplotlib.legend.Legend at 0x7feb45ae80d0>



[38]:
```
plt.plot(z, np.sin(np.pi * z), label="original")
plt.plot(z, estimate(20), '--', label=str(20))
plt.ylim(-1.2, 1.2)
plt.legend()
```

[38]: <matplotlib.legend.Legend at 0x7feb45a37d30>



[ ]: