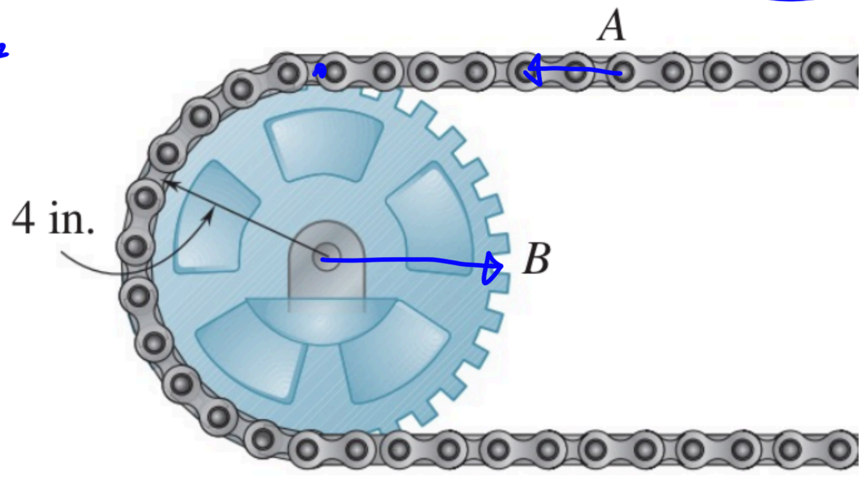
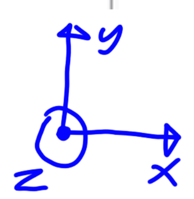
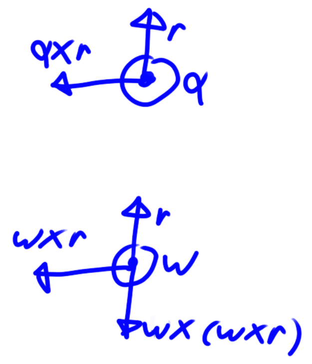
The sprocket wheel and chain shown are initially at rest. If the wheel has a uniform angular acceleration of 90 rad/s$^2$ counterclockwise, determine (*a*) the acceleration of point *A* of the chain, (*b*) the magnitude of the acceleration of point *B* of the wheel after 3 s.

$\vec{\alpha} = 90\,k \ \text{rad/s}^2$




$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$ $\qquad \vec{r} = 4j$ in




$\vec{a} = \vec{\alpha} \times \vec{r}$

$= \begin{vmatrix} i & j & k \\ 0 & 0 & 90 \\ 0 & 4 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 0 \\ 0 & 4 \end{matrix} = -4 \cdot 90\, i \ \text{in/s}^2$

$\boxed{= -360 \ \text{in/s}^2}$

$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha} t = \vec{0} + 90\,k \ \text{rad/s}^2 \ 3\,\text{s} = 270\,k \ \text{rad/s}$

$\vec{r} = 4i$ in

$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$\vec{\alpha} \times \vec{r} = \begin{vmatrix} i & j & k \\ 0 & 0 & 90 \\ 4 & 0 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 0 \\ 4 & 0 \end{matrix} = 90 \ \text{rad/s}^2 \ 4 \ \text{in}\ j$

$= 360\, j \ \text{in/s}^2$

$\vec{\omega} \times \vec{r} = \begin{vmatrix} i & j & k \\ 0 & 0 & 270 \\ 4 & 0 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 0 \\ 4 & 0 \end{matrix} = 270 \cdot 4\, j \ \text{in/s}$

$= 1080\, j \ \text{in/s}$

$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} i & j & k \\ 0 & 0 & 270 \\ 0 & 1080 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 0 \\ 0 & 1080 \end{matrix} = -270 \cdot 1080\, i \ \text{in/s}^2$

$= -2.92 \times 10^5\, i \ \text{in/s}^2$

$\vec{a} = 360\, j - 2.92 \times 10^5\, i \ \text{in/s}^2$

$|\vec{a}| = \sqrt{360^2 + (-2.92 \times 10^5)^2} \ \boxed{= 2.92 \times 10^5 \ \text{in/s}^2}$

$a_t = \alpha r = 90 \ \text{rad/s}^2 \ 4 \ \text{in} = 360 \ \text{in/s}^2$

$a_n = r\omega^2 = 4\,\text{in} (270 \ \text{rad/s})^2 = 2.92 \times 10^5 \ \text{in/s}^2$

$a = \sqrt{360^2 + (2.92 \times 10^5)^2} = 2.92 \times 10^5 \ \text{in/s}^2$