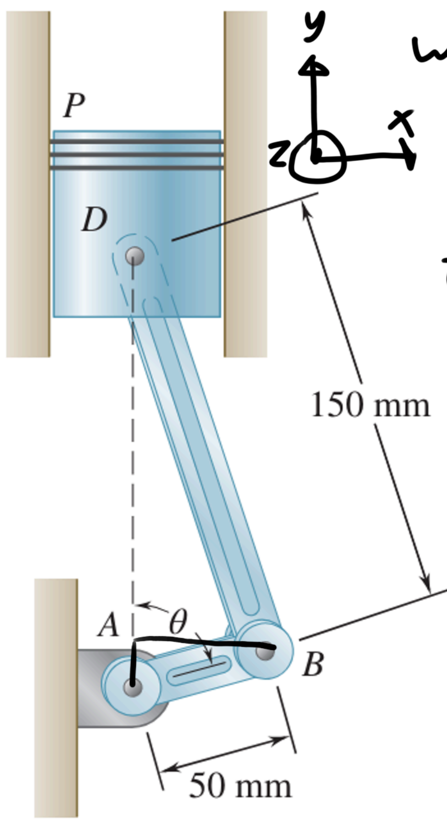
Knowing that crank AB rotates about point A with a constant angular velocity of 900 rpm clockwise, determine the acceleration of the piston P when $\theta = 60°$.



$\omega_{AB} = 900 \frac{rev}{min}\left(\frac{2\pi\, rad}{1\, rev}\right)\left(\frac{1\, min}{60\, s}\right)$

$= 94.2\ rad/s$

$\vec{\omega}_{AB} = -94.2\, k\ rad/s$

$\vec{a}_B = \cancel{\vec{a}_A} + \cancel{\vec{\alpha}} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$

$\vec{r}_{B/A} = 50\,mm \sin\theta\, i + 50\,mm \cos\theta\, j$

$= 43.3\, i + 25\, j\ \ mm$

$\vec{a}_B = -94.2^2(43.3\, i + 25\, j)$

$= -3.84\times10^5\, i - 2.22\times10^5\, j\ \ mm/s^2$

$\vec{a}_D = \vec{a}_B + \vec{\alpha}\times\vec{r}_{D/B} - \omega_{BD}^2\, \vec{r}_{D/B}$

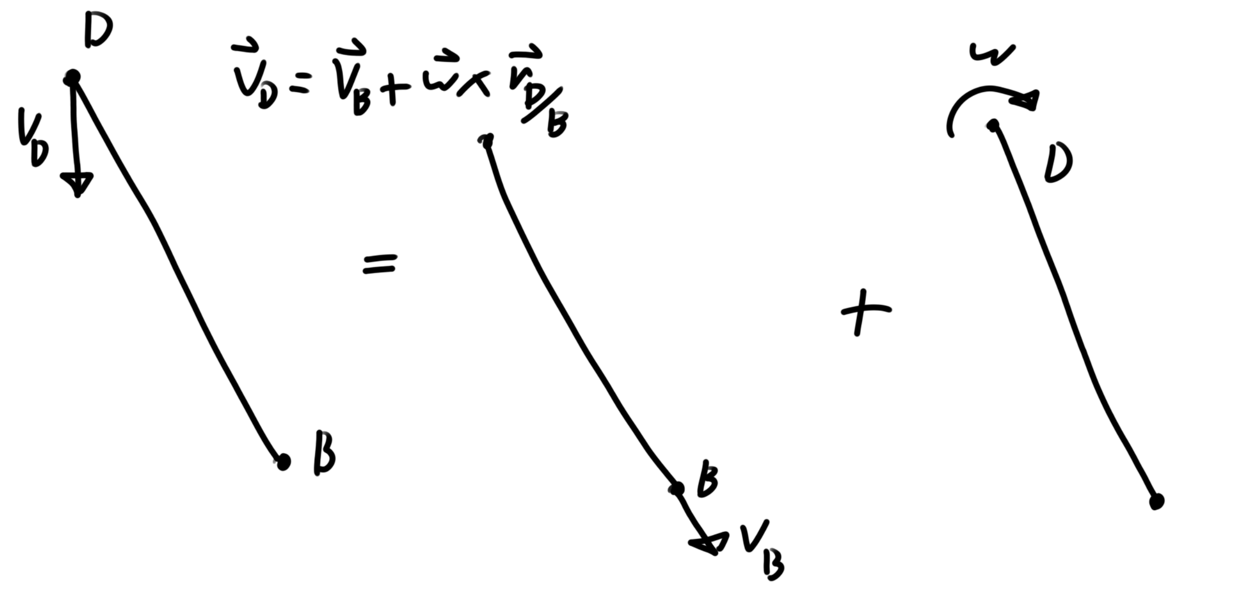
$\vec{V}_B = \cancel{\vec{V}_A} + \vec{\omega}\times\vec{r}_{B/A}$

$= \begin{vmatrix} i & j & k \\ 0 & 0 & -94.2 \\ 43.3 & 25 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 0 \\ 43.3 & 25 \end{matrix}$

$= 94.2\cdot 25\, i - 94.2\cdot 43.3\, j$

$= 2355\, i - 4079\, j\ \ mm/s$

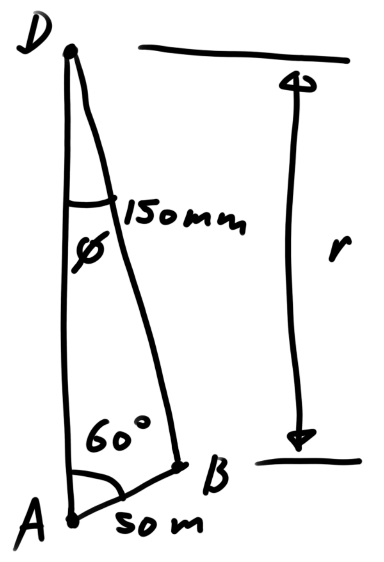
$\vec{V}_D = \vec{V}_B + \vec{\omega}\times\vec{r}_{D/B}$



$w = \frac{2355\ mm/s}{r}$

$= \frac{2355\ mm/s}{150\,mm \cos 16.3}$

$= 16.4\ rad/s$



$\vec{r}_{D/B} = 150\,mm \sin 16.3\, i + 150\,mm \cos 16.3\, i$

$= 43.4\, i + 143.6\, j\ \ mm$

$\frac{50\,mm}{\sin\phi} = \frac{150\,mm}{\sin 60}$

$\sin\phi = \sin 60 \frac{50\,mm}{150\,mm}$

$\phi = 16.8°$

$\vec{a}_D = \vec{a}_B + \alpha k\times\vec{r}_{D/B} - w^2\, \vec{r}_{D/B}$

$\alpha k \times \vec{r}_{D/B} = \begin{vmatrix} i & j & k \\ 0 & 0 & \alpha \\ 43.4 & 143.6 & 0 \end{vmatrix} \begin{matrix} i & j \\ 0 & 0 \\ 43.4 & 143.6 \end{matrix}$

$= -143.6\alpha\, i + 43.4\alpha\, j$

$a_D\, j = -3.84\times10^5\, i - 2.22\times10^5\, j - 143.6\alpha\, i + 43.4\alpha\, j - 16.4^2\, 43.4\, i$
$- 16.4^2\, 143.6\, j$

$a_D = -2.22\times10^5 + 43.4\alpha - 16.4^2\, 143.6$

$0 = -3.84\times10^5 - 143.6\alpha - 16.4^2\, 43.4$

$\boxed{\vec{a}_D = -1.48\times10^5\, j\ \ mm/s^2 = \vec{a}_P}$