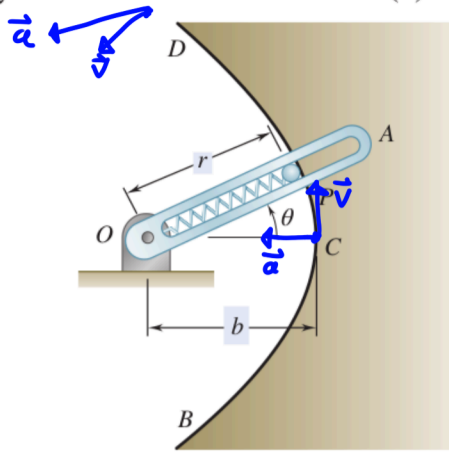


As rod OA rotates, pin P moves along the parabola BCD . Knowing that the equation of this parabola is $r = 2b/(1 + \cos \theta)$ and that $\theta = kt$, determine the velocity and acceleration of P when (a) $\theta = 0$, (b) $\theta = 90^\circ$.



$$\theta = kt \quad \frac{d\theta}{dt} = k \quad \frac{d^2\theta}{dt^2} = 0$$

$$r = \frac{2b}{1 + \cos \theta}$$

$$r = \frac{2b}{1 + \cos kt} \quad \frac{dr}{dt} = \frac{2bk \sin kt}{(1 + \cos kt)^2}$$

$$\frac{d^2r}{dt^2} = 2b \left(\frac{k^2 \cos kt}{(1 + \cos kt)^3} + \frac{2k^2 \sin^2 kt}{(1 + \cos kt)^3} \right)$$

$$\theta = 0 \quad kt = 0$$

$$r = \frac{2b}{1 + \cos 0} = \frac{2b}{1+1} = b$$

$$\frac{dr}{dt} = \frac{2bk \sin 0}{(1 + \cos 0)^2} = 0$$

$$\begin{aligned} \frac{d^2r}{dt^2} &= 2b \left(\frac{k^2 \cos 0}{(1 + \cos 0)^3} + \frac{2k^2 \sin^2 0}{(1 + \cos 0)^3} \right) \\ &= 2b \frac{k^2}{(1+1)^3} = \frac{2bk^2}{8} = \frac{bk^2}{4} \end{aligned}$$

$$\begin{aligned} \vec{v} &= \frac{dr}{dt} \vec{e}_r + r \frac{d\theta}{dt} \vec{e}_\theta \\ &= 0 \vec{e}_r + bk \vec{e}_\theta = bk \vec{e}_\theta \end{aligned}$$

$$\begin{aligned} \vec{a} &= \left(\frac{d^2r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right) \vec{e}_r + \left(r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \vec{e}_\theta \\ &= \left(\frac{bk^2}{4} - bk^2 \right) \vec{e}_r + (b \cdot 0 + 2 \cdot 0 \cdot k) \vec{e}_\theta \\ &= -\frac{3bk^2}{4} \vec{e}_r \end{aligned}$$

$$\theta = 90^\circ \quad kt = 90^\circ$$

$$r = \frac{2b}{1 + \cos 90^\circ} = \frac{2b}{1} = 2b$$

$$\frac{dr}{dt} = \frac{2bk \sin 90^\circ}{(1 + \cos 90^\circ)^2} = 2bk$$

$$\begin{aligned} \frac{d^2r}{dt^2} &= 2b \left(\frac{k^2 \cos 90^\circ}{(1 + \cos 90^\circ)^3} + \frac{2k^2 \sin^2 90^\circ}{(1 + \cos 90^\circ)^3} \right) \\ &= 2b \frac{2k^2}{1} = 4bk^2 \end{aligned}$$

$$\begin{aligned} \vec{v} &= \frac{dr}{dt} \vec{e}_r + r \frac{d\theta}{dt} \vec{e}_\theta \\ &= 2bk \vec{e}_r + 2bk \vec{e}_\theta \end{aligned}$$

$$\begin{aligned} \vec{a} &= \left(\frac{d^2r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 \right) \vec{e}_r + \left(r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} \right) \vec{e}_\theta \\ &= (4bk^2 - 2bk^2) \vec{e}_r + (2b \cdot 0 + 2 \cdot 2bk \cdot k) \vec{e}_\theta \\ &= 2bk^2 \vec{e}_r + 4bk^2 \vec{e}_\theta \end{aligned}$$