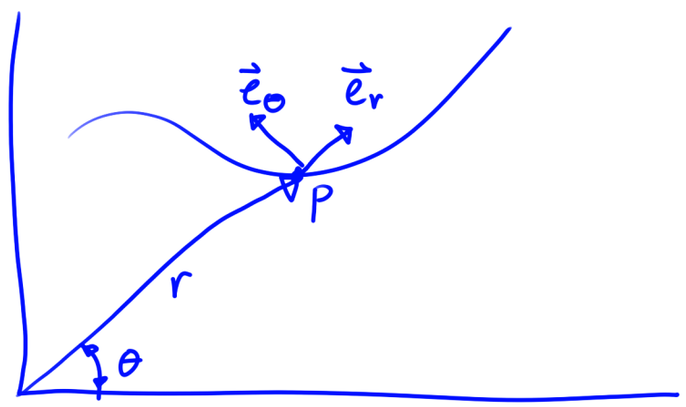
Radial and Transverse Components

$\vec{e}_r$ unit vector radial direction

$\vec{e}_\theta$ unit vector transverse direction



$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} \qquad \frac{d\vec{e}_\theta}{dt} = -\vec{e}_r\frac{d\theta}{dt}$$

$$= \vec{e}_\theta \frac{d\theta}{dt}$$

$$\vec{V} = \frac{d}{dt} r\vec{e}_r = \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt}$$

$$\boxed{= \frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta$$

$$= \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\vec{e}_r}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2}\vec{e}_\theta + r\frac{d\theta}{dt}\frac{d\vec{e}_\theta}{dt}$$

$$= \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\vec{e}_r$$

$$\boxed{= \left(\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right)\vec{e}_r + \left(r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt}\right)\vec{e}_\theta}$$