

## freq.bodesimp Bode plots for simple transfer functions

1 Although we have defined Bode plots in terms of the frequency response function  $H(j\omega)$ , it turns out that, due to its similarity, we can just as easily talk about the Bode plot of a transfer function. Since this is common convention, we proceed in kind.

2 It turns out that bode plots, both magnitude and phase, given their logarithmic scale (recall that the  $\omega$ -axes are also plotted logarithmically), are quite asymptotic to straight-lines for first- and second-order systems. Furthermore, higher-order system transfer functions can be re-written as the product of those of first- and second-order. For instance,

$$H(s) = \frac{\underline{M}_1 s + \underline{M}_2}{s^3 + \underline{M}_3 s^2 + \underline{M}_4 s + \underline{M}_5} \quad (1a)$$

$$= \underline{M}_1 \cdot (\underline{M}_2 s + 1) \cdot \frac{1}{\underline{M}_3 s + 1} \cdot \frac{1}{s^2 + \underline{M}_4 s + \underline{M}_5} \quad (1b)$$

3 Recall (from, for instance, phasor representation) that for products of complex numbers, phases  $\phi_i$  add and magnitudes  $M_i$  multiply. For instance,

$$M_1 \angle \phi_1 \cdot \frac{1}{M_2 \angle \phi_2} \cdot \frac{1}{M_3 \angle \phi_3} = \frac{M_1}{M_2 M_3} \angle (\phi_1 - \phi_2 - \phi_3). \quad (2)$$

And if one takes the logarithm of the magnitudes, they add; for instance,

$$\log \frac{M_1}{M_2 M_3} = \log M_1 - \log M_2 - \log M_3. \quad (3)$$

There is only one more link in the chain: first- and second-order Bode plots depend on a handful of parameters that can be found directly from transfer functions. There is no need to compute  $|H(j\omega_0)|$  and  $\angle H(j\omega_0)$ !

4 In a manner similar to [Example freq.bode-1](#), we construct Bode plots for several simple transfer functions in this lecture. Once we have

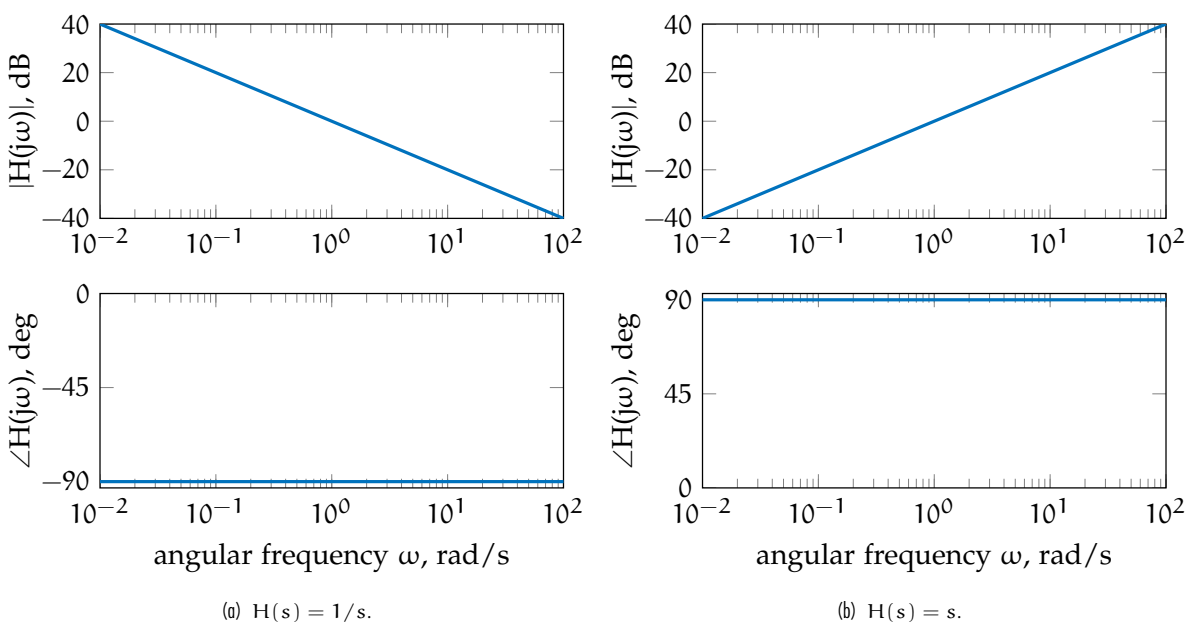
these simple “building blocks,” we will be able to construct sketches of higher-order systems by graphical addition because logarithmic magnitudes and phases combine by summation, as shown in [Lec. freq.bodesketch](#).

### Constant gain

5 For a transfer function that is simply a constant real gain  $H(s) = K$ , the frequency response function is trivially  $H(j\omega) = K$ . Its magnitude  $|H(j\omega)| = |K|$ . For positive gain  $K$ , the phase is  $\angle H(j\omega) = 0$ , and for negative  $K$ , the phase is  $\angle H(j\omega) = 180$  deg.

### Pole and zero at the origin

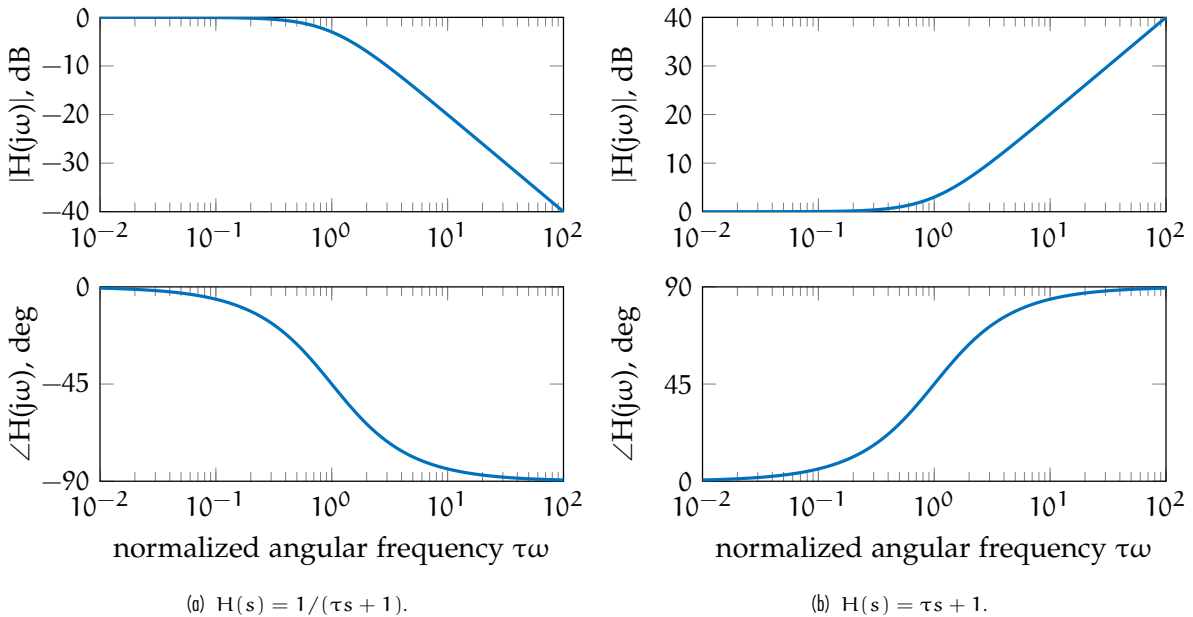
6 In [Example freq.bode-1](#), we have already demonstrated how to derive from the transfer function  $H(s) = s$ , a zero at the origin, the frequency response function plotted in [Fig. bodesimp.1](#). Similarly, for  $H(s) = 1/s$ , a pole at the origin, the frequency response function plotted in [Fig. bodesimp.1](#).



**Figure bodesimp.1:** Bode plots for (a) a pole at the origin and (b) a zero at the origin.

## Real pole and real zero

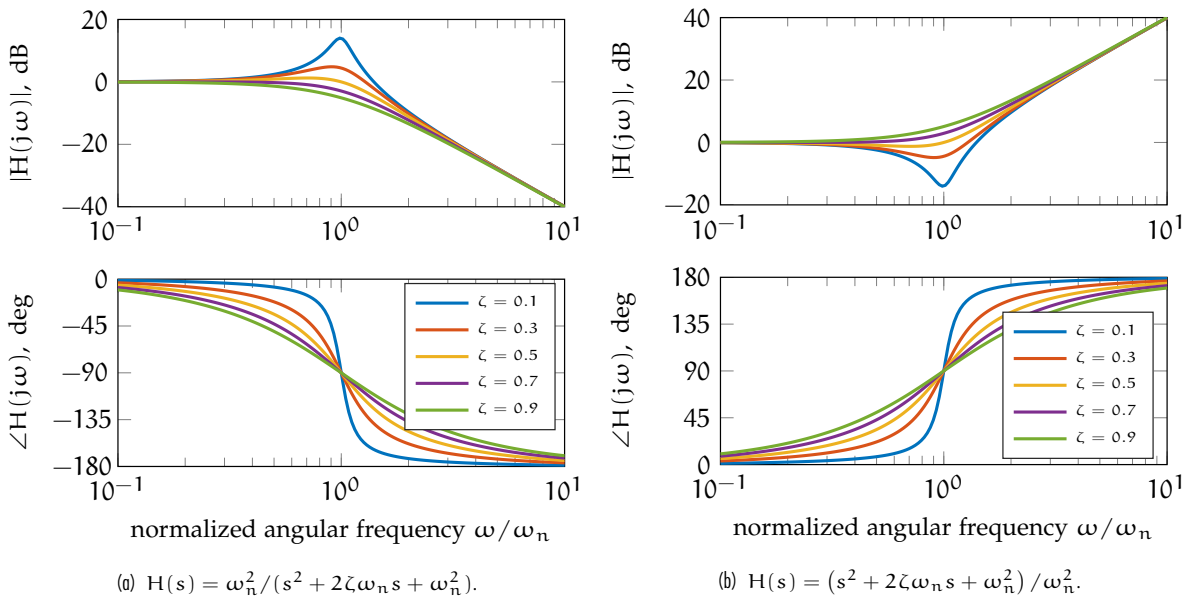
7 The derivations for real poles and zeros are not included, but the resulting Bode plots are shown in Fig. bodesimp.2.



**Figure bodesimp.2:** Bode plots for (a) a single real pole and (b) a single real zero.

## Complex conjugate pole pairs and zero pairs

8 The derivations for complex conjugate pole pairs and zero pairs are not included, but the resulting Bode plots are shown in Fig. bodesimp.3.



**Figure bodesimp.3:** Bode plots for (a) a complex conjugate pole pair and (b) a complex conjugate zero pair.