freq.nystab Stability from the Nyquist plot

The Nyquist stability criterion has established closed-loop stability for unity controller gain. Of course, we would like to learn about the stability for any value of gain K and closed-loop transfer function

$$T(s) = \frac{G(s)}{1 + KG(s)H(s)}. \tag{1}$$

The Nyquist plot is of $G(\Gamma_N)H(\Gamma_N)$, and if we include the gain it becomes $KG(\Gamma_N)H(\Gamma_N)$. This simply scales the magnitude by K, which "stretches" the image by a factor of K. Recall that, for stability, we are concerned with encirclements of -1. So scaling K can change the number of encirclements.
For instance, let's consider a system with open-loop transfer function

$$G(s)H(s) = \frac{(s-1)(s-2)}{(s+3)(s+5)}. \tag{2}$$

This system has $P = 0$ open-loop poles in the right-half-plane, thus for no encirclements of -1, the closed-loop system is stable by the Nyquist criterion. Fig. nystab.1 shows the Nyquist plot for three different values of gain K:

($K = 1.00$) this is the Nyquist plot we have thus far considered, and $N = 0$, so $Z = 0$ and the closed-loop system is stable;

($K = 4.00$) this stretches the previous plot by a factor of 4, $N = -2$, so $Z = +2$ and the closed-loop system is unstable;

($K = 2.67$) scales the original plot such that it intersects -1, at which point the system is marginally stable.

Stability from the positive $j\omega$-axis image, alone

It turns out we can determine stability from the image of the positive $j\omega$-axis, alone. Due to the

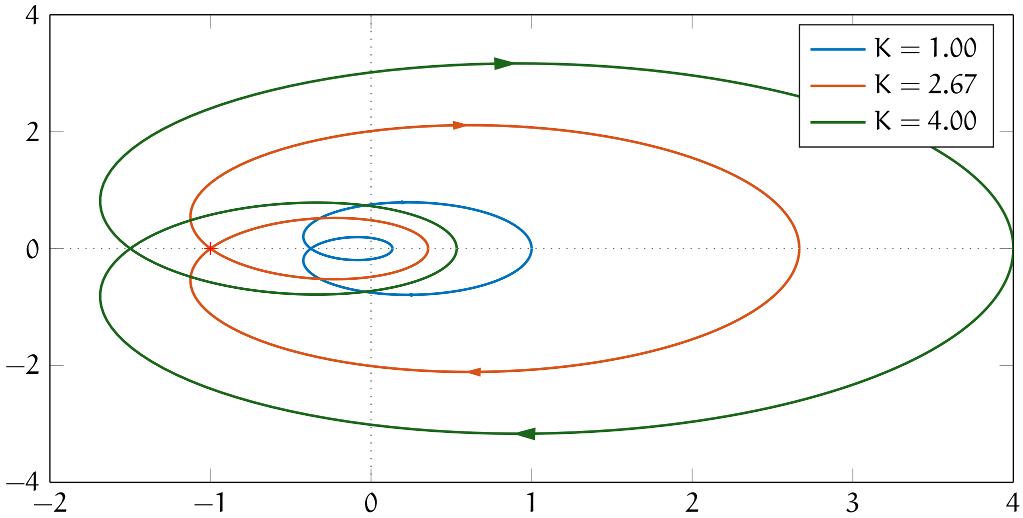

Figure nystab.1:

real-axis symmetry of the poles and zeros of transfer functions, the image of the negative $j\omega$-axis is simply a reflection about the real axis of the image of the positive $j\omega$-axis. Nothing that occurs at zero magnitude affects stability. The same is true for that which occurs at infinite magnitude, which never intersects the negative real axis.[1]

1. This is true for poles and zeros on the $j\omega$ axis as long as we draw the infinitesimal detour into the right-half-plane, as we are accustomed to doing.

Gain margin and phase margin

Let us consider a system with a Nyquist plot for which smaller values of gain K yield a stable closed-loop system and higher values of K that yield one that is unstable. It turns out that these types of systems are very common. We know that the key to stability in such systems is the number of encirclements of -1. We are now ready to define two key quantities, the gain margin and the phase margin. For a given gain K, these parameters quantify "how stable" a given system is. The gain margin G_M is a logarithm of the distance from -1 to the Nyquist plot's negative real-axis intercept $-1/a$, as shown in Fig. nystab.2. Specifically, in dB

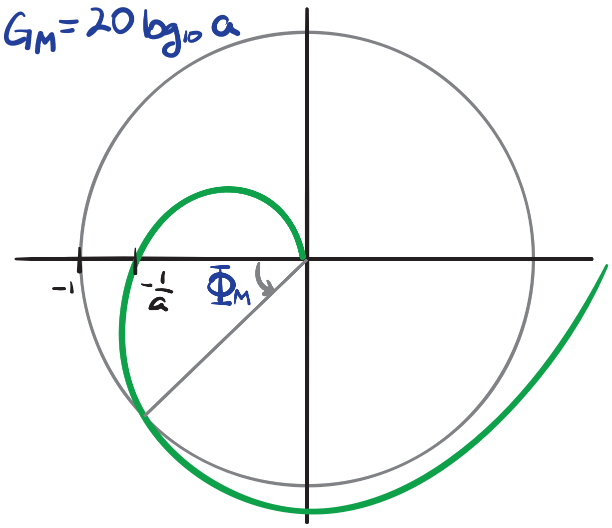

Figure nystab.2:

(the typical manner of expressing G_M),

$$G_M = 20 \log_{10} a. \tag{3}$$

The phase margin Φ_M is defined as the angle at which the magnitude is unity as the Nyquist plot approaches its negative real-axis intercept $-1/a$. This is typically difficult to find, mathematically, from a Nyquist plot. Primarily for this reason, in a moment we will switch to a Bode plot representation, in which both G_M and Φ_M are easily found.

As already mentioned, G_M and Φ_M can be considered measures of stability. We can consider this to mean that higher G_M and Φ_M correspond to greater confidence that the closed-loop system will remain stable, even under small changes to its system parameters.