

B.01 Controllability, observability, and stabilizability

The three topics controllability, observability, and stabilizability are three topics of central concern to linear systems theory.

Controllability

Controllability is defined as follows.

Definition B.1: controllable and uncontrollable

If there exists some input to a linear system such that any initial state in its state space can be evolved in finite time to any final state in its state space, the system is controllable. Otherwise, the system is uncontrollable.

A given system's controllability can be determined from the following.

Definition B.2: controllability matrix

Let a linear system of order n and number of inputs r have state space $\{A, B, C, D\}$. We define the $n \times nr$ controllability matrix to be

$$\mathcal{U} = [B \mid AB \mid A^2B \mid \dots \mid A^{n-1}B].$$

The following well-known theorem, left unproven here, allows us to easily determine the controllability of a given system.

Theorem B.3: controllability

A linear system is controllable if its controllability matrix has full rank. If it is less than full rank, the linear system is uncontrollable.