intro.sdet State–determined systems

1 A system is defined to be a collection of objects and their relations contained within a boundary. The collection of those objects that are external to the system and yet interact with it is called the environment. System variables are variables that represent the behavior of the system, both those that are internal to the system and those that are external—that is, with the system's environment.

system
boundary
environment
system variables

2 There are three important classes of system variable, all typically expressed as vector-valued functions of time t, conventionally all expressed as column-vectors (and called "vectors" even though they're vector-valued functions …because nothing makes sense and we're all going to die). Consider Figure sdet.1 for the following definitions. Input variables are system variables that do not depend on the internal dynamics of the system; for a system with r input variables, the "input vector" is a vector-valued function $\mathbf{u} : \mathbb{R} \to \mathbb{R}^r$. The environment prescribes inputs, making them independent variables. Conversely, output variables are system variables of interest to the designer; for a system with m output variables, the "output vector" is a vector-valued function $\mathbf{y} : \mathbb{R} \to \mathbb{R}^m$. Outputs may or may not directly interact with the environment. Finally, a minimal set of variables that define the internal state of the system are defined as the state variables; for a system with n state variables, the "state vector" is a vector-valued function $\mathbf{x} : \mathbb{R} \to \mathbb{R}^n$.

input variables
input vector
output variables
output vector
state variables
state vector

3 We consider a very common class of system: those that are state-determined, which are those for which (Derek Rowell and David N. Wormley. System Dynamics: An Introduction. Prentice Hall, 1997)

state–determined

1. a mathematical description,
2. the state at time t_0, called the initial condition $x(t)|_{t=t_0}$, and
3. the input u for all time $t \geqslant t_0$

initial condition

are necessary and sufficient conditions to determine $x(t)$ (and therefore $y(t)$) for all $t \geqslant t_0$.

4 The "mathematical description" of the system requires a set of primitive elements be assigned to represent its internal and external interactions. The equations derive from two key types of mathematical relationships:

1. the input-output behavior of each primitive element and
2. the topology of interconnections among elements.

The former generate elemental equations and the latter, continuity or compatibility equations.

elemental equations
continuity equations
compatibility equations

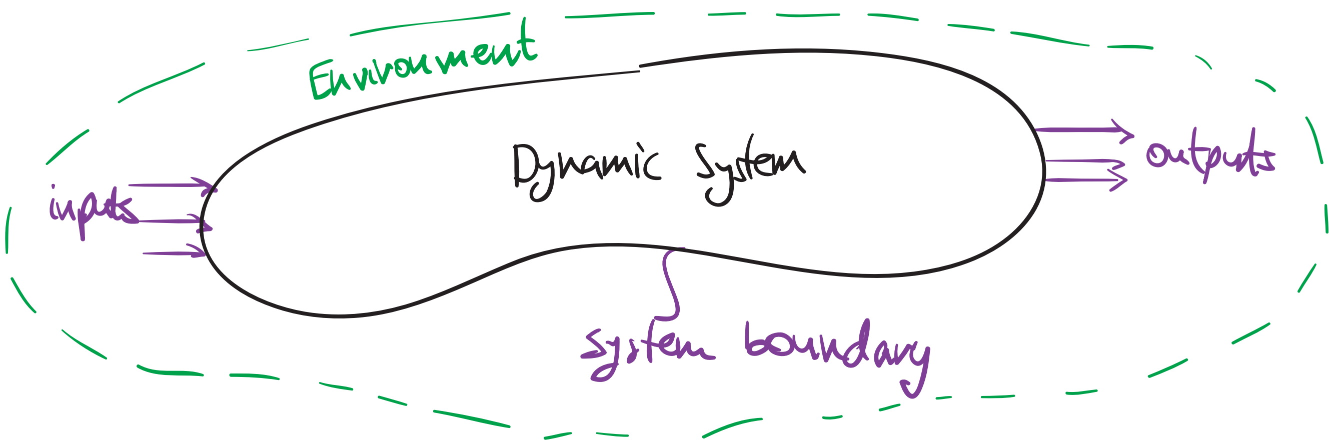

Figure sdet.1: illustration of a system and its environment.

Example intro.sdet–1 **re: a state–determined system**

In the RC circuit shown, let V_s be a source and v_o the voltage of interest. Identify

1. the system boundary,
2. the input vector,
3. the output vector,
4. a state vector,
5. an elemental equation,
6. and which equations might be continuity or compatibility equations.