intro.lump Energy, power, and lumping

law of energy conservation

1 The law of energy conservation states that, for an isolated system, the total energy remains constant. Let $\mathcal{E} : \mathbb{R} \to \mathbb{R}$ be the function of time representing the total energy in a system and $\mathcal{P} : \mathbb{R} \to \mathbb{R}$ be the function of time representing power into the system, defined as

power

$$\mathcal{P}(t) = \frac{d\mathcal{E}(t)}{dt}. \tag{1}$$

The energy in a system can change if it exchanges energy with its environment. We consider this exchange to occur through a finite number of ports, each of which can supplies or removes energy (positive or negative power), as in Figure lump.1. This is expressed in an equation for power into a port $\mathcal{P}_i$ and N ports as

We construct our systems such that they have no internal energy sources.

Lumping

2 We have assumed power enters a system via a finite number of ports. Similarly, we assume the energy in a system is stored in a finite number M of distinct elements with energy $\mathcal{E}_i$ such that

$$\mathcal{E}(t) = \sum_{i=1}^{M} \mathcal{E}_i(t). \tag{2}$$

We call these elements energy storage elements. Energy can also be dissipated from the system via certain elements called energy dissipative elements.

energy storage elements

energy dissipative elements

3 Considering a system to have a finite number of elements, as we have done, requires

abstraction

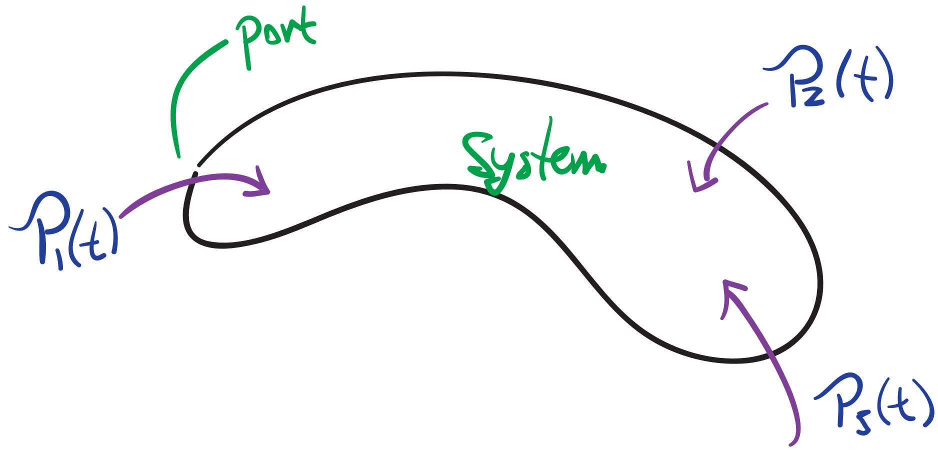

Figure lump.1: system ports

a specific kind of abstraction from real systems. A familiar example is the "point mass" of elementary mechanics. We say it interacts with its environment via specific connections called ports (maybe it's attached to a spring element) and behaves a certain way in these interactions (for a mass element, Newton's second law). We do not often encounter an object that behaves as if it has mass, but no volume. Yet, this is a useful abstraction for many problems.

4 When we abstract like this, considering an object to be described fully as a discrete object with interaction ports, we are said to be lumped-parameter modeling. This is often contrasted with distributed or continuous modeling, which consider the element in greater detail. For instance, an object might be considered to be distributed through space and perhaps be flexible or behave as a fluid.

lumped−parameter modeling
distributed modeling
continuous modeling

5 It is lumped-parameter modeling because we typically define a parameter that governs the behavior of the element, such as resistance or mass. This parameter will enter the system's dynamics via an elemental equation such as Ohm's Law in the case of a resistor or Newton's Second Law in the case of a mass.

elemental equation

6 Determining if lumped-parameter modeling

is proper for a given system is dependent on the type of insight one wants to acheive about the system. The system itself does not prescribe the proper modeling technique, but the desired understanding does. Every system is incredibly complex in its behavior, if one considers it at a fine-granularity. In this light, it is striking that simple models work at all. Nevertheless, lumping is highly effective for many analyses.

7 It is important to note that lumped-parameter models can be applied at different levels of granularity for the same system. Finite element modeling can use a large number of small lumped-parameter model elements to approximate a continuous model. Such applications are beyond the scope of this course.

finite element modeling