intro.mecht Mechanical translational elements

1 We now introduce a few lumped-parameter elements for mechanical systems in translational (i.e. straight-line) motion. Newton's laws of motion can be applied. Let a force f and velocity v be input to a port in a mechanical translational element. Since, for mechanical systems, the power into the element is

force

velocity

$$\mathcal{P}(t) = f(t)v(t) \tag{1}$$

we call f and v the power-flow variables for mechanical translational systems. Some mechanical translational elements have two distinct locations between which its velocity is defined (e.g. the velocity across a spring's two ends) and other elements have just one (e.g. a point-mass), the velocity of which must have an inertial frame of reference. This is analogous to how a point in a circuit can be said to have a voltage—by which we mean "relative to ground." In fact, we call this mechanical translational inertial reference ground.

power–flow variables

ground

2 Work done on the system over the time interval $[0, T]$ is defined as

work

$$W \equiv \int_0^T \mathcal{P}(\tau)d\tau. \tag{2}$$

Therefore, the work done on a mechanical system is

$$W = \int_0^T f(\tau)v(\tau)d\tau. \tag{3}$$

3 The linear displacement x is

linear displacement

$$x(t) = \int_0^t v(\tau)d\tau + x(0). \tag{4}$$

Similarly, the linear momentum is

linear momentum

$$p(t) = \int_0^t f(\tau)d\tau + p(0). \tag{5}$$

energy storage elements

energy dissipative elements

4 We now consider two elements that can store energy, called energy storage elements; an element that can dissipate energy to a system's environment, called an energy dissipative element; and two elements that can supply power from outside a system, called source elements.

source elements

Translational springs

5 A translational spring is defined as an element for which the displacement x across it is a monotonic function of the force f through it. A linear translational spring is a spring for which Hooke's law applies; that is, for which

translational spring

linear translational spring

$$f(t) = kx(t), \tag{6}$$

where f is the force through the spring and x is the displacement across the spring, minus its unstretched length, and k is called the spring constant and is typically a function of the material properties and geometry of the element. This is called the element's constitutive equation because it constitutes what it means to be a spring.

spring constant

constitutive equation

6 Although there are many examples of nonlinear springs, we can often use a linear model for analysis in some operating regime. The elemental equation for a linear spring can be found by time-differentiating Equation 6 to obtain

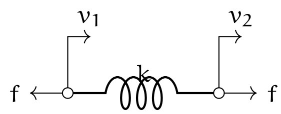

Figure mecht.1: schematic symbol for a spring with spring constant k and velocity drop $v = v_1 - v_2$.

elemental equation

We call this the elemental equation because it relates the element's power-flow variables f and v.

7 A spring stores energy as elastic potential energy, making it an energy storage element. The amount of energy it stores depends on the

force it applies. For a linear spring,

$$\mathcal{E}(t) = \frac{1}{2k} f(t)^2. \tag{7}$$

Point-masses

8 A non-relativistic translational point-mass element with mass m, velocity v (relative to an inertial reference frame), and momentum p has the constitutive equation

$$p = mv. \tag{8}$$

Once again, time-differentiating the constitutive equation gives us the elemental equation:

which is just Newton's second law.

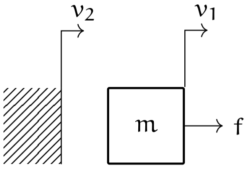

Figure mecht.2: schematic symbol for a point-mass with mass m and velocity drop $v = v_1 - v_2$, where v_2 is the constant reference velocity.

9 Point-masses can store energy (making them energy storage elements) in gravitational potential energy or, as will be much more useful in our analyses, in kinetic energy

$$\mathcal{E}(t) = \frac{1}{2} m v^2. \tag{9}$$

Dampers

10 Dampers are defined as elements for which the force f through the element is a monotonic function of the velocity v across it. Linear dampers have constitutive equation (and, it turns out, elemental equation)

dampers

linear dampers

$$f = Bv \tag{10}$$

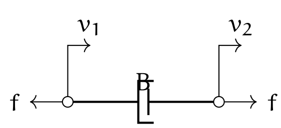

Figure mecht.3: schematic symbol for a damper with damping coefficient B and velocity drop $v = v_1 - v_2$.

where B is called the damping coefficient. Linear damping is often called viscous damping because systems that push viscous fluid through small orifices or those that have lubricated sliding are well-approximated by this equation. For this reason, a damper is typically schematically depicted as a dashpot.

damping coefficient

viscous damping

dashpot

11 Linear damping is a reasonable approximation of lubricated sliding, but it is rather poor for dry friction or Coulomb friction, forces for which are not very velocity-dependent. Aerodynamic drag is quite velocity-dependent, but is rather nonlinear, often represented as

dry friction

Coulomb friction

drag

where c is called the drag constant.

12 Dampers dissipate energy from the system (typically to heat), making them energy dissipative elements.

Force and velocity sources

ideal force source

13 An ideal force source is an element that provides arbitrary energy to a system via an independent (of the system) force. The corresponding velocity across the element depends on the system.

ideal velocity source

14 An ideal velocity source is an element that provides arbitrary energy to a system via an independent (of the system) velocity. The corresponding force through the element depends on the system.